Inventory partial selling quantity model of ameliorating items with linear price dependent demand

Biswajit Mandal[1], Asoke Kumar Bhunia[2] and Manoranjan Maiti[3]

[1]Department of Mathematics, Raja N. L Khan Womens' College, Midnapore, India.
[2]Department of Mathematics, The University of Burdwan, Burdwan, India.
[3]Department of Applied Mathematics, Vidyasagar University, Midnapore-721102, India.

Abstract:

This paper deals with the development of an inventory model for ameliorating items. Generally, fast growing animals like duck, pigs, broiler etc. in poultry farm, highbred fishes in bhery (pond) etc. are these type of items. When these ameliorating items are kept/ preserved at farm and bhery, the weight or value or utility of items increases due to growth and also decreases due to disease, death or some other factors. As the increase or decrease of stock is considered in weights and the rate of amelioration is greater than that of deterioration, it is clear that the stocks will be increased due to combined effect of both amelioration and deterioration. This paper investigates an instantaneous replenishment inventory model for above type of items under profit maximization. Due to the combined effect of amelioration and deterioration of items, the resulting rate i.e. the rate of net amelioration (in weights) follows the weibull distribution over time. Here, the selling rate is assumed to be a deterministic function of selling price of an item and initially it is less than the rate of net amelioration. Rate of net amelioration slowly decreases with time and after sometime, it becomes less than the selling rate. Shortages are not allowed. Finally, the model is illustrated with the help of a numerical example.

AMS Mathematics Subject Classification Code: 90B05

Keywords: Amelioration, Partial-selling and Weibull-distribution.

Introduction :

The classical inventory models have been developed under the assumption that the life time of an item is infinite while it is in storage. This means that an item once in stock remains physically unchanged and fully usable for satisfying the demand during their storage period. Generally, MRO (maintenance, repair and operating supplies) items, infrastructural items etc. are of this type. Again due to inadequate preservation facilities of items like paddy, wheat or any other food grains, vegetables, fruits, drugs, fishes etc., a certain fraction of these is either damaged or decayed or vapourised or effected by some other factors during their storage period and these are not in perfect condition to satisfy the customer's demand. More over, the partially effected (due to deterioration) units of some items like fruits, vegetables, fishes etc. are sold at a reduced price. A good number of inventory models have been developed in this area. To get an idea of the trends of recent research, one may refer to the works of Dutta and Pal [1991], Giri et.al [1996b], Goyal and Gunesekaran [1997b], Bhunia and Maiti [1997a], Sarkar et.al [1997e], Kar et.al [2001] and others. In recent years, some inventory models [Padnanebhan and Vrat [1995a], Abad [1994, 1996b] and Luo [1998] have been developed for perishable items like perfumes, medicines, blood etc. which have the fixed life time and can't be used after the expired date. Except deteriorating or perishable items, there are another type of items made of glass, ceramic etc. which are stored one after another in the form of heaped stock and break/get damaged due to the accumulated stress. Recently, Mondal and Maiti [1997d] developed some inventory models for such products.

In the existing literature, O.R. scientists or practitioners did not give much attention for fast growing animals like broiler, ducks, pigs etc. in the poultry farm, highbred fishes in bhery(pond)which are known as ameliorating items . When these items are in storage, the stock increases (in weight) due to growth of the items and also decreases due to death, various diseases or some other factors. Till now, only Hwang [1997c,1999b] reported this type of inventory model .In 1997 , first Hwang [1997c] studied two inventory models like EOQ model and partial selling quantity (PSQ) model under the assumption that the time to ameliorating of an item follows the Weibull distribution. Again, in 1999, Hwang [9] developed inventory models for both ameliorating and deteriorating items separately under the issuing policies, LIFO (Last input Fast output) and FIFO (First input First output).

Therefore, in view of changing the items either in weight or in quantity, the general inventory system can classified as follows:

- **Non-decaying inventory system**

  [Conventional inventory system]
- **Decaying inventory system with full deterioration**

  [In this case, the deteriorated units can't be used]
- **Decaying inventory system with partial deterioration**

  [In this case, the deteriorated units are sold with a reduced price]
- **Ameliorating inventory system**

In the present competitive market, the effect of marketing policies and conditions such as the price variations and advertisement of an item changes its selling rate amongst the public. In selecting of an item for use, the selling price of an item is one of the decisive factors to the customers. It is commonly seen that lesser selling price causes increase in the selling rate whereas higher selling price has the reverse effect. Hence, the selling rate of an item is dependent on the selling price of that item. This selling rate function must be a decreasing function with respect to the selling price. Incorporating the price variations, recently several researchers i.e Urban [1992], Ladany and Sternleib [1974], Subramanyam and Kumaraswamy [1981], Goyal and Gunasekaran [1997b], Bhunia and Maiti [1997a], Luo [1998], Weng [1995b] and Das et.al [1999a] developed their models for deteriorating and non - deteriorating items .

In Hwang[8], partial selling quantity model is developed where a surplus amount due to amelioration effect are sold after each cycle length for infinite time horizon . This means that the rate of amelioration is greater than the selling rate of that item. But it is not always true. For the ameliorating items, the rate of amelioration will slowly decrease over time. As the rate of deterioration is less than that of amelioration, so the resulting rate i.e. the rate of net amelioration follows the Weibull distribution over time in a decreasing nature. But initially, it is greater than the selling rate. Incorporating this situation, we have developed a partial selling inventory model for ameliorating items under profit maximization. The selling rate is a deterministic function on the selling price of an item. Shortages are not allowed. Finally, a numerical example is used to study the behaviour of the model.

Assumption and notations:-

The proposed mathematical model of inventory replenishment problem is developed under the following assumptions and notations **:**

i) The inventory system involves only one item and one stocking point.

ii) The time horizon is infinite.

iii) Replenishments are instantaneous with a known constant lead-time.

iv) Shortages are not permitted.

v) Amelioration occur when the item is effectively in stock.

vi) T is the cycle length.

vii) The initial stock level is S.

viii) t_i , (i= 1,2,3,……,n) be the time of partial selling point i.e., the time point for which the surplus amount S_0 will be sold.

ix) The cycle length of accumulation of S_0 units in any interval is proportional to the previous.

x) The inventory carrying cost C_h per unit per unit time, the replenishment cost C_o per replenishment, the cost of amelioration, C_a per unit, the purchase cost, C_p per unit are known and constant.

xi) The demand rate R (p) is dependent on the selling price p per unit.

xii) α , β be the parameters of the Weibull distribution whose probability density function is

$$f(t) = \alpha\beta\ t^{\beta-1}\exp(-\alpha\beta\, t^{\beta-1})$$

The instantaneous rate of net amelioration of the on hand inventory in any time t is A (t) which obeys the Weibull distribution

$$A(t) = \frac{f(t)}{1-F(t)} = \alpha\beta\, t^{\beta-1}$$

where α, $\beta > 0$ and F(t) is the distribution function of Weibull distribution.

As the amelioration rate decreases over time, the cycle length of net amelioration of S_0 units will be greater than that of previous. Hence, according to the assumption (ix) we can write

$$t_i - t_{i-1} = k(t_{i-1} - t_{i-2}) \tag{1}$$

where k is independent of t_i's and greater than unity.

From (1) one can write

$$t_i = \frac{k^i - 1}{k-1} t_1 \quad (i = 1,2,3,\ldots\ldots\ldots,n)$$

The Mathematical Model:

In this problem, we assume that initially the stock level is S. Due to higher rate of net amelioration and relatively lower rate of selling, stock level reaches at $S+S_0$ at $t = t_1$. Then the excess S_0 amounts are sold at a time and the inventory level will be dropped to S. This process will be continued for the intervals (t_1, t_2), (t_2, t_3) & so on up to $t = t_n$. At $t = t_n$, the rate of net amelioration is equal to the demand rate and then stock will be decreased. At t= T, it will be zero. Let q(t) be the inventory level at any time t in $t_{i-1} \le t \le t_i$ (i= 1,2,3,…..,n). The inventory is increased due to net amelioration and is depleted mainly due to selling. Hence the rate of change of stock at any instant t is equal to the algebraic sum of the selling rate and net amelioration at any instant t ($t_{i-1} \le t \le t_i$) (see fig.-1).

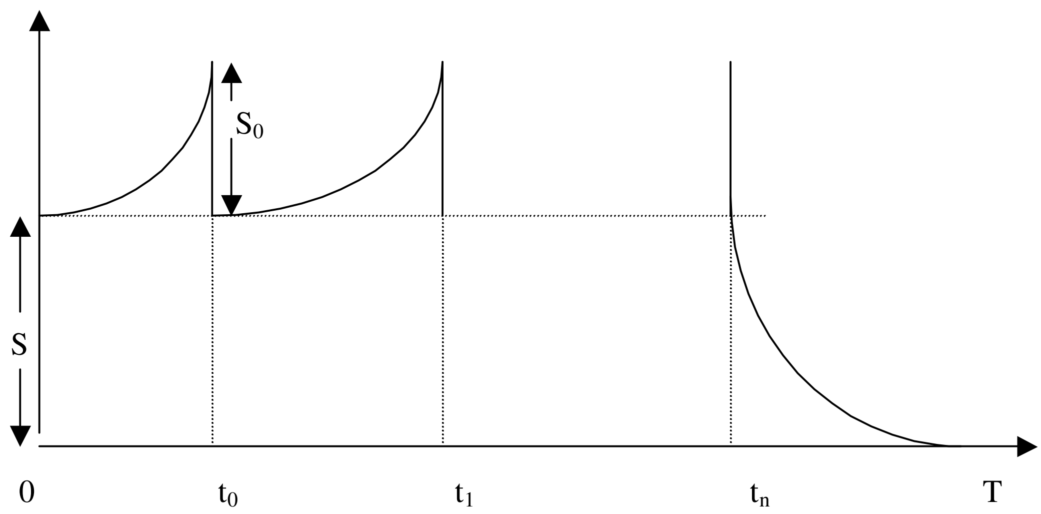

Fig. –1 : Inventory situation for the model

Therefore, the inventory level q(t) satisfies the following differential equations:

$$dq(t)/dt = A(t)q(t) - R(p) \qquad t \in [\, t_{i-1} \,,\, t_i \,] \quad i = 1,2,3,..n \tag{2}$$

with boundary condition $q(t) = S$ at $t = t_{i-1}$

$$i = 1,2,3,..n \tag{3}$$

Further $q(t) = S + S_o$ at $t = t_i$ $i = 1,2,3\ldots n$ (4)

Again, in $t_n \leq t \leq T$, let the stock level q(t), at any time t, which satisfies the following differential equation

$$dq(t) / dt = A(t)\, q(t) - R(p) \qquad t \in [\, t_n \,,\, T] \tag{5}$$

with boundary conditions

$$q(t) = S \qquad \text{at} \quad t = t_n \tag{6}$$

and

$$q(t) = 0 \qquad \text{at} \quad t = T \tag{7}$$

Hence, using (3), we get the solution of (2) as

$$q(t) = S\exp(\alpha(t^{\beta} - t_{i-1}^{\beta-1})) - R\exp(\alpha t^{\beta}) \int_{t_{i-1}}^{t} \exp(\alpha t^{\beta})\, dt \qquad i = 1,2,3,\ldots,n \tag{8}$$

Now using (3), we have

$$S + S_0 = S\exp(\alpha(t_i^{\beta} - t_{i-1}^{\beta})) - R\exp(\alpha t_i^{\beta}) \int_{t_{i-1}}^{t_i} \exp(-\alpha t^{\beta})\, dt \qquad i = 1,2,3,...,n \qquad (9)$$

For i= 1 we have

$$S_0 = S(\exp(\alpha t_1^{\beta}) - 1) - R\exp(\alpha t_1^{\beta}) \int_0^{t} \exp(-\alpha t^{\beta})\, dt \qquad (10)$$

Using (6), we get the solution of (5) as follows:

$$q(t) = S\exp(\alpha(t^{\beta} - t_n^{\beta})) - R\exp(\alpha t^{\beta}) \int_{t_n}^{T} \exp(-\alpha t^{\beta})\, dt \qquad (11)$$

Now using (7), we have

$$S = R\exp(\alpha t_n^{\beta}) \int_{t_n}^{T} \exp(-\alpha t^{\beta})\, dt \qquad (12)$$

We assume that A_i is the total ameliorating items with in the interval $[t_{i-1}, t_i]$ and $R(t_i - t_{i-1})$ is the total demand with in the interval $[t_{i-1}, t_i]$. So by our assumption A_i satisfy relation

$$A_i - R(t_i - t_{i-1}) = S_0 \qquad (13)$$

$$\text{i.e.} \quad A_i = S_0 + R(t_i - t_{i-1}) \qquad i = 1,2,3,..,n \qquad (14)$$

Let A' is the total net amelioration of items within the interval $[t_n, T]$

Hence, $A' + S = R(T - t_n)$

$$A' = R(T - t_n) - S \qquad (15)$$

Now, that total units of net amelioration items over the cycle is given by

$$A = \sum_{i=1}^{n} A_i + A' = \sum_{i=1}^{n} S_0 + \sum_{i=1}^{n} R(t_i - t_{i-1}) + R(T - t_n) - S$$

$$= nS_0 + RT - S \qquad (16)$$

The inventory amount in $[t_{i-1}, t_i]$ and in $[t_n, T]$ is given by

$$H_i = \int_{t_{i-1}}^{t_i} q(t)\, dt = S \int_{t_{i-1}}^{t_i} \exp(\alpha(t^{\beta} - t_n^{\beta}))\, dt - R \int_{t_{i-1}}^{t_i} \int_{t_{i-1}}^{t} \exp(\alpha t^{\beta} - \alpha \tau^{\beta})\, d\tau\, dt \tag{17}$$

$$i = 1, 2, 3, \ldots\ldots n$$

and

$$H' = \int_{t_n}^{T} q(t)\, dt = S \int_{t_n}^{T} \exp(\alpha(t^{\beta} - t_n^{\beta}))\, dt - R \int_{t_n}^{T} \int_{t_n}^{t} \exp(\alpha t^{\beta} - \alpha \tau^{\beta})\, d\tau\, dt \tag{18}$$

$$\textit{Hence total holding } \mathrm{cost}\ (H) = c_h \left(\sum_{i=1}^{n} H_i + H'\right) \tag{19}$$

As the selling rate is equal to the net ameliorating rate at $t = t_n$

So, $\quad \alpha\beta\, t_n^{\beta-1} S = R$

i.e. $$\alpha\beta\, t_{t_n}^{\beta-1} \int_{t_n}^{T} \exp(-\alpha(t^{\beta} - t_n^{\beta}))dt = 1 \tag{20}$$

We take $$G(n, t_1, T, k) = \alpha\beta\, t_n^{\beta-1} \int_{t_n}^{T} \exp(-\alpha t^{\beta})dt - 1 \tag{21}$$

Profit per cycle = (Sales revenue per cycle) - (total cost per cycle)
And Total cost per cycle = Purchase cost per cycle + ordering cost + inventory carring cost + additional cost for growth of items.

The net profit for the entire inventory system is the difference between the sales revenue and the total cost of the system

$$\begin{aligned} X(n, t_1, T, k) &= nS_0p + pRT - CpS - C_a(\textstyle\sum A_i + A') - C_h(\textstyle\sum H_i + H') - C_0 \\ &= nS_0p + pRT - C_pS - C_a(nS_0 + RT - S) - C_h(\textstyle\sum H_i + H') - C_0 \end{aligned} \tag{22}$$

Therefore, for the fixed value of p, thr profit function π (n, t_1, T, k) (average profit per unit time for the entire replenishment cycle) of the inventory system is given by

$$\pi(n, t_1, T, k) = X / T$$

Here the profit function is a function of three continuous variables t_1,T,k and one discrete variable n.

Our objective is to find out the optimal values of t_1,T and the average profit by maximizing the profit function for a fixed n. Therefore, we have to maximize π (n, t_1, T,

k) with respect to T,k, and t_1 for a fixed n subject to the condition (21). So we use the Langrange's multiplier method for the function

$$F = \pi\,(n, t_1, T, k) + \lambda G(n, t_1, T, k) \tag{23}$$

For the given values of p and n, the necessary conditions for π to be maximize are

$$\frac{\partial F}{\partial t_1} = 0, \quad \frac{\partial F}{\partial k} = 0, \quad \frac{\partial F}{\partial T} = 0 \quad and \quad \frac{\partial F}{\partial \lambda} = 0 \tag{24}$$

Theorem – I: For the fixed value of n, the replenishment schedule satisfying the first order optimality conditions (24) satisfies the second order conditions for a maximum.

Proof: The above theorem can be proved easily by showing the associated Hassian matrix of the solution of the equations (23) have alternatively positive and negative principle minors of order one two and three.

On simplification, from (23) we have

$$\frac{\partial \pi}{\partial k}\frac{\partial G}{\partial t_1} - \frac{\partial \pi}{\partial t_1}\frac{\partial G}{\partial k} = 0, \tag{25}$$

$$(T\frac{\partial \pi}{\partial T} - X)\frac{\partial G}{\partial k} - T\frac{\partial G}{\partial T}\frac{\partial X}{\partial k} = 0, \tag{26}$$

$$G\,(n, t_1, T, k) = 0, \tag{27}$$

The above equations (25) - (27) are in general nonlinear equation for t_1 , k , T, and can be solved when the explicit form of R(p) is specified. A closed form solution cannot be obtained for those variables . Only numerical solutions can be derived by using a suitable numerical methods (such as N- R method, Iteration method etc.) for the specified form of R(p) and a fixed n.

Numerical Examples:

As an illustration of the developed model, let us consider a hypothetical system with the following parameter:

C_0 = \$ 150 / order, a = 200, b = 0.1, C_p = \$ 15/ unit, C_h = \$ 1/ unit, C_a = \$ 6/ unit, p = \$20/unit, R (p) = a-bp (a > 0, b ≥ 0), $\alpha = 0.9$ and $\beta = 0.9$.

For the fixed value of p, solving the nonlinear equations (25) – (27) by Newton-Raphson method, we get the optimal values of n, t_1 , T, k and the average profit π (n, t_1, T, k) are as follows :

n = 21, t_1 = 0.48, t_n = 10.51, T = 19.36, k = 1.01, π = \$10,104.13

Concluding Remarks :

In the present paper, we have formulated and solved a deterministic inventory model for ameliorating items considering partial selling. Here it is assumed that wholesalers and retailers purchase this type of life stocks in a lot and sell the units at a uniform rate for a certain period of time. During this period, the stocks increase in weight due to amelioration and at the end of the time cycle, the stock is brought down to the size of initial stock through one time sale. At the end, when the net amelioration rate is very less, then the total stock is cleared. This model can be applied to determine the optimal inventory policy for the life stocks, which exhibit the characteristics of amelioration consider in this model.

In the assumptions and notations section, β is assumed as a proper fraction (i.e. $0 < \beta < 1$). This indicates that the rate of net amelioration due to the combined effect of amelioration and deterioration is a decreasing function of time t. Generally, the amelioration and deterioration is initially high and it decreases over time for the ameliorating items. Again the selling rate must be a decreasing function with respect to p as the lower price causes the higher selling rate and vice versa. In the numerical example, the selling rate is taken as the linear function of p.

In the developing countries like India Pakistan, Nepal etc., the proposed model has very importance as the culture of life stocks has been taken now-a-days in a very large scale to develop the economic condition as well as for the self employment.

References

ABAD, P. L. (1994), Supplier pricing and lot-sizing when demand is price sensitive**,** **Euro. J. Opl. Res**., 78, 334-354.

ABAD, P. L. (1996a), Optimal pricing and lot-sizing under conditions of perishability and partial back-ordering, **Mgmt. Sc.,** 42, 1093-1104.

BHUNIA, A. K. and MAITI, M. (1997a), An inventory model for decaying items with selling price, frequency of advertisement and linearly time-dependent demand with shortages, **IAPQR Transactions**, 22, 41-49.

DATTA, T. K. and PAL, A. K. (1991), A finite-horizon model for inventory returns and special sales of deteriorating items with shortages, **Asia-Pacific J. Opl. Res**., 8, 179-188.

DAS, K , BHUNIA, A. K. and MAITI, M. (1999a), An inventory model for deteriorating items with shortages and selling price-dependent demand , **IAPQR Transactions.**, 24, 65-72.

GIRI, B.C., PAL, S., GOSWAMI, A. AND CHAUDHURI, K. S. (1996b), An inventory model for deteriorating items with stock-dependent demand rate, **Euro. J. Opl. Res**., 95, 604 - 610.

GOYAL, S. K. and GUNASEKARAN, A. (1997b), An integrated production - inventory - marketing model for deteriorating items, **Computers Ind. Engg**., 28, 41-49.

HWANG, H. S. (1997c), A study on an inventory model for items with Weibull ameliorating, **Computers Ind. Engg.,** 33, 701-704.

HWANG, H. S. (1999b), Inventory models for both deteriorating and ameliorating items, **Computers Ind. Engg.,** 37, 257 - 260.

KAR, S, BHUNIA, A. K. and MAITI, M. (2001), Inventory of multi-deteriorating items sold from two shops under single management with constraints on space and investment, **Comp.&Oper. Res.,** 28, 1203-1221.

LADANY, S. AND STERNLEIB A. (1974), The intersection of economic ordering quantities and marketing policies, **AIIE Transations**., 6, 35 - 40 .

LUO, W. (1998), An integrated inventory system for perishable goods with backordering , **Computers Ind. Engg**., 34, 685-693.

MANDAL, B. N. and PHAUJDER, S. (1989), An inventory model for deteriorating items and stock-dependent consumption rate, **J. Opl. Res. Soc.,** 40, 483-488.

MANDAL, M. and MAITI, M. (1997d), Inventory model for damagable items with stock-dependent demand and shortages, **Opsearch**, 34, 156-166.

MUKHERJEE, S. P. (1987), Optimum ordering interval for time varying decay rate of inventory, **Opsearch**, 24, 19 - 24.

PADMANABHAN, G. and VRAT, P. (1995a), EOQ models for perishable items under stock-dependent selling rate, **Euro. J. Opl. Res**., 281-292.

SARKAR , B. R, MUKHERJEE S. and BALAN , C. V, (1997e), An order-level lot-size inventory model with inventory-level-dependent demand and deterioration, **Inter. J. Prod. Econ**., 48, 227 - 236.

SUBRAMANYM, S. and KUMARASWAMY, S. (1981), EOQ formula under varying marketing policies and conditions, **AIIE Transations.,** 13**,** 312 - 314 **.**

URBAN, T. L. (1992), Deterministic inventory models incorporating marketing decisions, **Computers & Ind. Engg.,** 22, 85-93.

WENG, K. Z. (1995b), Modelling quantity discounts under general price-sensitive demand functions: Optimal policies and relations, **Euro. J. Res. Soc.,** 86, 300-314.