Special issue on “Graph Theory and its Applications”

Editorial

It is my great pleasure to accept Guest Editorial offer from Professor Neculai Andrei for this special issue of ***Advanced Modeling and Optimization*** which marks fifteen years of publication. The journey of this journal started from 1999, and now it becomes a truly international one. This journal covers a wide range of subjects by authors from almost all countries of the world. This special issue of *Advanced Modeling and Optimization* is devoted to the very old but still young subject *graph theory*. The concept of this subject is currently used to solve many real life problems, viz. internet, mobile computing, social networks, etc.

The origin of graph theory was started with the problem of Königsberg bridge, in 1735. The paper written by Leonhard Euler on the Seven Bridges of Königsberg and published in 1736 is regarded as the first paper in the history of graph theory. This problem leads to the concept of Eulerian Graph. Euler studied the problem of Königsberg bridge and represented it as a pictorial diagram called Eulerian graph. More than hundred years of Euler’s work, Cayley introduced a special type of graph, known as trees. The concept of tree, was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in electrical networks or circuits. The structural properties of chemical molecule have investigated using the concept of trees. Cayley linked his results on trees with the contemporary studies of chemical composition. In particular, the term "graph" was introduced by Sylvester in a paper published in 1878 in Nature. In 1852, Thomas Gutherie found the famous four colour problem. A computer-aided proof is given in 1976 by Kenneth Appel and Wolfgang Haken. The proof involved checking the properties of 1,936 configurations by computer, and was not fully accepted at the time due to its complexity. A simpler proof considering only 633 configurations was given twenty years later by Robertson, Seymour, Sanders and Thomas.

The concepts of graph theory are widely used to study and model various applications in different areas of sciences, technologies even in social sciences. This concept is also widely used in Operations Research to solve, the traveling salesman problem, the shortest path, shortest spanning tree, job scheduling problem, transportation problem, activity networks, theory of games, study of molecules, construction of bonds in chemical compounds..Using the game theoretic technique several multi-strategic problems are solved in engineering, economics and war science to find optimal way to perform certain tasks in competitive environments. The study of DNA sequence is also investigated using the concept of graph theory. The social networks such as Facebook, ResearchGate, Orkut, LikedIn, etc. are constructed using graphs and analysis on these networks are based on the analysis of the corresponding graphs. Two prominent applications of graphs are frequency assignment in cellular networks and map labeling or colouring.

Due to the above mentioned applications till today the graph theory is important for the scientific community and we feel that a special issue should be published on “Graph Theory and its Applications”. In this special issue 11 articles have been accepted for publication. We hope that the readers of Advanced Modeling and Optimization will find in these papers stimulating ideas and useful results. The summaries of these papers in this issue are listed below:

The first article entitled “Strongly indexable graphs: some new perspectives” is an invited paper. In this paper, the authors expound the connection of strong indexers of graphs with Sidon sequences and Fibonacci sequences and explores classes of strongly indexable chain graphs whose blocks are complete or, the so-called `Husimi trees'.

In the second article, the T-Hypohamiltonian graph of non-Hamiltonian graph is defined. To find the cyclic path covering number of hypohamiltonian graphs, two theorems have been developed and based on these results the cyclic path covering number of all non Hamiltonian graphs are found.

In the third paper, the authors deal with *L(0,1)*-labelling problem on interval graphs. *L(0,1)*-labelling of a graph $G=(V,E)$ is a function f from the vertex set $V(G)$ to the set of non-negative integers such that adjacent vertices get number zero apart, and vertices at distance two get distinct numbers. The goal of *L(0,1)*-labelling problem is to produce a legal labelling that minimize the largest label used. This problem is NP-complete for general graph. In this paper, this problem is investigated for interval graph and an efficient algorithm for finding the *L(0,1)*-labelling of the said graph is presented.

In the fourth paper, bounds for Neighbourhood resolving number for sum and composition of two graphs are obtained. Neighbourhood resolving number for Mycielski graphs is discussed. Also nice results involving neighbourhood resolving numbers of a graph and its complement are obtained.

In fifth paper, the authors defined v_k-cyclic path covering to find the cyclic path covering number of a digraph in which some of the vertices have privilege of being interior vertex of more than one path and they find v_k-cyclic path covering number of simple digraph and have defined $\{v_{k1}, v_{k2}, v_{k3}\ldots v_{km}\}$-cyclic path covering and have found the $\{v_{k1}, v_{k2}, v_{k3},\ldots,v_{km}\}$-cyclic path covering number of such coverings. Also we defined e_m-cyclic path covering number and finally found the $\{e_m,v_k\}$-cyclic path covering number of any digraph.

In sixth paper, the authors have established bounds on the fractional out-domination number for the generalised Kautz digraph and presented a condition for the fractional out-domination number attaining its lower bound. They also obtain the exact value of the fractional out-domination number for Kautz digraph.

The seventh article of this issue has concentrated on decomposition of total graphs. All the vertex-vertex adjacency, vertex-edge incidence and edge-edge incidence relations are considered in the formation of the total graph. For a finite simple connected graph G, T(G) can be decomposed into G and complete subgraphs of order equal to the degrees of each of the vertices in G. Also, T(G) can be decomposed into disjoint union of L(G) and q copies of C_3, where q is the size of G, i.e. T(G(p,q)) =L(G)UqC_3.

The eighth paper is devoted to the study of cordial labelling of cactus graph. Suppose $G=(V,E)$ be a graph with vertex set V and edge set E. A vertex labelling $f:V\rightarrow\{0,1\}$ induces an edge labelling $f^*:E\rightarrow\{0,1\}$. For $i\in\{0,1\}$, let $v_f(i)$ and $e_f(i)$ be the number of vertices v and edges e with $f(v)=i$ and $f^*(e)=i$ respectively. A graph is cordial if there exists a vertex labelling f such that $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$. In this paper, the authors label the vertices of cactus graph by cordial labelling and have shown that cactus graph is cordial under some restrictions.

In the ninth paper, the authors are investigated some new families of edge product cordial graph.

The tenth paper is also devoted to study of a particular type of graph labeling called skolem difference mean labeling. A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from 1,2…p+q in such a way that the edge $e = uv$ is labeled with $\frac{|f(u)-f(v)|}{2}$ if $|f(u) - f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u) - f(v)|$ is odd and the resulting labels of the edges are distinct and are from 1,2…q. A graph that admits skolem difference mean labeling is called skolem difference mean graph. In this paper, the authors study the skolem difference mean labeling of the graphs $C_{2k+1}\otimes K_{1,m}$ and $C_{2k}\otimes K_{1,m}$.

In eleventh paper, a complete graph structure based comparison sorting algorithm, has been proposed that takes $\Theta(n^2)$ time in the worst-case, where n is the number of records in the given list to be sorted.

I would like to take this opportunity to express my sincere thanks to all the authors for their contributions to this special issue, the reviewers for their valuable input, insight, and expert comments, and the Editor-in-Chief, Professor Neculai Andrei, for his strong support and suggestion in the preparation of the final presentation of putting together this special issue.

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