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A Markovian Feedback Queue with Retention of Reneged Customers

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Abstract

We consider a single server, finite capacity Markovian feedback queue with reneging and retention of reneged customers in which the inter-arrival and service times follow exponential distribution. The reneging times are assumed to be exponentially distributed. Feedback in queueing literature represents customer dissatisfaction because of inappropriate quality of service. In case of feedback, after getting partial or incomplete service, customer retries for service. After the completion of service,each customer may rejoin the system as a feedback customer for receiving another regular service with probability p_1 or he can leave the system with probability q_1 where $p_1 + q_1 = 1$. A reneged customer can be retained in many cases by employing certain convincing mechanisms to stay in queue for completion of service. Thus, a reneged customer can be retained in the queuing system with some probability q_2 or he may leave the queue without receiving service with probability $p_2(= 1 - q_2)$.

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The steady-state solution of the model is obtained iteratively. Some performance measures are also derived. Finally, some important queuing models are derived as special cases of this model.

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1 Introduction

Feedback in queueing literature represents customer dissatisfaction because of inappropriate quality of service. In case of feedback, after getting partial or incomplete service, customer retries for service. In computer communication,the transmission of protocol data unit is sometimes repeated due to occurrence of an error. This usually happens because of non-satisfactory quality of service. Rework in industrial operations is also an example of a queue with feedback. [Takacs,1948] studies queue with feedback to determine the stationary process for the queue size and the first two moments of the distribution function of the total time spent in the system by a customer. [Davignon and Disney, 1973] study single server queues with state dependent feedback.[Santhakumaran and Thangaraj, 2000] consider a single server feedback queue with impatient and feedback customers. They study $M/M/1$ queueing model for queue length at arrival epochs and obtain result for stationary distribution, mean and variance of queue length. [Thangaraj and Vanitha, 2009] obtain transient solution of $M/M/1$ feedback queue with catastrophes using continued fractions. The steady-state solution, moments under steady state and busy period analysis are calculated. [Ayyappan et al., 2010] study $M/M/1$ retrial queueing system with loss and feedback under non-pre-emptive priority service by matrix geometric method.

The notion of customer impatience appear in the queuing theory in the work of [Haight,1957]. He considers a model of balking for M/M/1 queue in which there is a greatest queue length at which an arrival will not balk. This length is a random variable whose distribution is same for all customers. [Haight,1959] studies a queue with reneging in which he studies the problem like how to make rational decision while waiting in the queue, the probable effect of this decision etc.[Ancker and Gafarian,1963] study $M/M/1/N$ queuing system with balking and reneging and perform its steady-state analysis. [Ancker and Gafarian,1963] also obtain results for a pure balking system (no reneging) by setting the reneging parameter equal to zero. [Kumar and Sharma, 2012a] study $M/M/1/N$ queuing system with retention of reneged customers. They obtain steady-state solution and study the effect of probability of customer retention on various performance measures. [Kumar and Sharma, 2012b] study $M/M/1/N$ queuing system with retention of reneged customers and balking. They obtain steady-state solution and perform sensitivity analysis of the model.

In this paper, we consider a single server, finite capacity Markovian feedback queuing system with reneging and retention of reneged customers in which the inter-arrival and service times follow exponential distribution. The reneging times are assumed to be exponentially distributed. After the completion of service,each customer may rejoin the system as a feedback customer for receiving another regular service with probability p_1 or he can leave the system with probability q_1 where $p_1 + q_1 = 1$. A reneged customer can be retained in many cases by employing certain convincing mechanisms to stay in the queue for completion of his service. Thus, a reneged customer can be retained in the queuing system with some probability q_2 and may leave the queue without receiving service with probability $p_2 = 1-q_2$. This effort to retain the reneging customer in the queue for his service has positive effect on business of any firm.

Rest of the paper is arranged as follows: In section 2, the model is described. Scetion 3 deals with performance measures of the system, and in section 4, special cases of the model are discussed. The paper is concluded in section 5.

2 Model Description

Consider an $M/M/1/N$ queue with instantaneous Bernoulli feedback with reneged customers and retention of reneged customers. Capacity of the system is taken as finite. Customers arrive at the service station one by one according to Poisson stream with arrival rate λ. There is a single server which provides service to all arriving customers. Service times are independently and identically distributed exponential random variables with parameter μ. After completion of each service, the customer can either join at the end of the queue with probability p_1 or he can leave the system with probability q_1 where $p_1 + q_1 = 1$. The customers both newly arrived and those that are fed back are served in order in which they join the tail of original queue. We do not distinguish between the regular arrival and feedback arrival. The customers are served according to first come, first served rule. The customer in queue (regular arrival or feedback arrival) may become impatient when the service is not available for a long time. In fact, each customer, upon arrival, activates an individual timer, which follows an exponential distribution with parameterξ. This time is reneging time of an individual customer after which customer either decide to abandon the queue with probability $p_2(= 1 - q_2)$ or never to return with complimentary probability. A system of difference differential equations satisfied by the $M/M/1/N$ feedback queue with reneged customers and retention of reneged customers are modelled as a continuous time Markov chain (CTMC).

Let $\{X(t) : t \in \Re^+\}$ be the number of customers in the system at time t. Let $P_n(t) = P(X(t) = n), n = 0, 1, 2, N$ be the state probabilities that there are n customers in the system at time t. Based on above assumptions the Chapman-Kolmogorov forward differential- difference equations of the model are

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu q_1 P_1(t) \tag{1}$$

$$\frac{dp_n(t)}{dt} = -[\lambda+\mu q_1+(n-1)\xi p_2]P_n(t)+(\mu q_1+n\xi p_2)P_{n+1}(t)+\lambda P_{n-1}(t) \quad 1 \le n \le N-1 \tag{2}$$

$$\frac{dP_N(t)}{dt} = \lambda P_{N-1}(t) - [\mu q_1 + (N-1)\xi p_2]P_N(t) \quad n = N \tag{3}$$

In steady state $\lim_{t\to\infty} P_n(t) = P_n$ and therefore $\frac{dP_n(t)}{dt} = 0$ as $t \to \infty$ Thus , the steady state equations corresponding to (1)-(3) are as follow

$$0 = -\lambda P_0 + \mu q_1 P_1 \tag{4}$$

$$0 = -[\lambda+\mu q_1+(n-1)\xi p_2]P_n+(\mu q_1+n\xi p_2)P_{n+1}+\lambda P_{n-1} \quad 1 \le n \le N-1 \tag{5}$$

$$0 = \lambda P_{N-1} - [\mu q_1 + (N-1)\xi p_2]P_N \quad n = N \tag{6}$$

Solving iteratively equations (4)-(6), we get

$$P_n = \prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2} P_0 \tag{7}$$

Using the normalization condition, $\sum\limits_{n=0}^{N} P_n = 1$,we get

$$P_0 = \frac{1}{1 + \sum_{n=1}^{N} \prod_{k=1}^{n} \frac{\lambda}{\mu q_1+(k-1)\xi p_2}} \tag{8}$$

3 Performance Measures

In this section, some important performance measures are derived.

(i) The Expected System Size (L_s)

$$L_s = \sum_{n=1}^{N} n[\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2}]P_o \tag{9}$$

(ii) The Expected Queue Length (L_q)

$$L_q = \Big[\sum_{n=1}^{N} n[\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2}]P_0 - \frac{\lambda}{\mu q_1}\Big] \tag{10}$$

(iii) The Expected Waiting Time in the System (W_s)

$$W_s = \Big[\frac{1}{\lambda}\sum_{n=1}^{N} n[\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2}] - \frac{\lambda}{\mu q_1}\Big]P_0 \tag{11}$$

(iv) The Expected Waiting Time in the queue (W_q)

$$W_q = \Big[\frac{1}{\lambda}\sum_{n=1}^{N} n[\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2}]P_0 - \frac{1}{\mu q_1}\Big] \tag{12}$$

(v) The Expected Number of customers Served, $E(\text{Customer Served})$

$$E(\text{Customer Served}) = \sum_{n=1}^{N} n\mu q_1 P_n$$

$$E(\text{Customer Served}) = \sum_{n=1}^{N} n\mu q_1 [\prod_{k=1}^{n} \frac{\lambda}{\mu q_1 + (k-1)\xi p_2}] P_0 \tag{13}$$

Where P_0 is computed in (8).

4 Special Cases

1 When $q_1 = 1, 0 < q_2 < 1$, the queueing system reduces to $M/M/1/N$ queuing model with reneging and retention of reneged customers as studied by [Kumar and Sharma, 2012a].

2 When $q_2 = 0, 0 < q_1 < 1$, the queueing system reduces to $M/M/1/N$ queuing model with feedback and reneging.

3 When $q_2 = 1, i.e. \xi = 0, 0 < q_1 < 1$, the queueing system reduces to $M/M/1/N$ queuing model with feedback.

4 when $q_2 = 1, i.e. \xi = 0, q_1 = 1$ The queueing system reduces to $M/M/1/N$ queuing model

5 Conlusions

In this paper, we study a finite capacity single-server Markovian feedback queuing model with reneging and retention of reneged customers. The steady-state solution of the model is obtained and some measures of performance are also derived. The model results may be useful in modeling various production and service processes involving feedback and impatient customers. The model analysis is limited to finite capacity. The infinite capacity case of the model can also be studied. Further, the model can be solved in transient state to get time-dependent results.

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