# Quadratic Programming Method to Solve the Non-linear Optimization Problems Applied to EMS

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***Abstract*— The subject of optimization applied to the practical real life problems of electrical energy management system (EMS) is a complex mixture of modeling, mathematical formulation, algorithmic solution processes and in the end the application of the optimal result to the process, where the process, which should be optimized, must be analyzed and must be understood in great depth. In addition complex mathematical equations and algorithms can be involved; also, computer know-how and software engineering capabilities must be present. In the end, the optimal result must be applied to the process. In this paper text we will give a combination of standard mathematical optimization problem formulations together with a straightforward solution procedures based on a quadratic programming algorithm to solve economic dispatch problem as example of EMS .**

***Keywords-component; Optimization, Algorithms, linear programming, Quadratic programming, energy management system .***

## I. 1. Introduction

In the widest sense of the word, optimization is the process of choosing rationally among given alternatives. Most real-world optimization problems (OP) are far too complex or stochastic to be analyzed or solved using mathematics. There are, however, important problems for which one can give a mathematical description, which is both tractable, that is, can be "solved" in some sense, and is a good enough approximation of the problem one really wants to solve. Some typical examples are problems of scheduling (machines, aircraft carriers, trains), dimensioning (of pipes, power plants, inventory size), routing (salesmen, wire, telephone calls), mixing (animal feed, petroleum, products), and construction (bridges, airplanes, integrated circuits) [1]-[5].

As an area of mathematics, optimization is the theory of minimizing or maximizing a function, over some feasible set [1] [6]. Depending on the properties of the involved function and sets, the field of optimization is divided into several sub fields. During the Second World War, researchers tried to formulate and analyze various mathematical models for transportation of goods, production planning and allocation of scarce resources. This was the birth of operations research. Since then the theory and application of operations research has been rapidly developing. The main quantitative tools of operations research are mathematical programming and simulation [7] [8]. In mathematical programming one is mainly concerned with theory and algorithms for optimization in finite-dimensional spaces. A good survey over the different topics in mathematical programming is given in [9], and for some history see [10].

## II. Mathematical Optimization

A system defined in terms of $m$ equations and $n$ unknown variables can be divided into three fundamental types of problems:

1. If $m = n$ the problem is an *algebraic* problem and usually has at least one solution.
2. If $m > n$ the problem is *over constrained* and cannot be solved in general.
3. If $m < n$ the problem is *under constrained* and many solutions can exist that satisfy the system requirements.

The third category of OP is the one discussed in this paper.

Variables in the context of optimization represent the individual elements that uniquely define the OP being considered. The variables can be divided into several categories:

1. Known variables: These variables have usually fixed numerical values. They cannot be solved for because they are known beforehand. Mathematically these variables can be seen as constant numerical values or as parameters.

2. Unknown variables: These are the interesting variables; the goal is to find a set of numeric values for these unknown variables such that optimality is achieved. Within the set of unknown variables two main subcategories can be found:

*Control variables*: These unknown variables represent individual elements which can be directly controlled within the process to be optimized.



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*State variables*: These unknown variables cannot be controlled directly within the process to be optimized. Their value is a consequence of the control variables choice and how the process reacts to these control variables values.

Thus an OP consists of different types of variables and usually an under constrained equation system. At this point nothing has been said about the term 'best' or 'improving' the current solution.

Obviously the term "goal" or "objective" of a process must be defined. One wants to minimize or maximize some objective. The objective is defined as a function of variables. In order to compare the optimality of the many solutions of the under constrained system, the objective function allows to give a merit to each solution.

*A. Mathematical formulation and optimality conditions* [2]

The general OP formulation is summarized as follows with:
$n$ = number of variables;
$m$ = number of equality constraints (linear or nonlinear)
$p$ = number of inequality constraints (linear or nonlinear)

$$\begin{aligned} &\min \quad F(x) \\ &\text{Subject to} \\ &g_i(\mathrm{x})=0, \text{ for } i=1,\dots,m \\ &h_i(\mathrm{x})\le 0 \text{ , for } i=1,\dots,p \end{aligned} \tag{1}$$

The problem is to determine the set of values of the vector (x) for which all equality constraints $g$(x) = 0 and all inequality constraints $h$(x) = 0 are satisfied and for which the objective function is at a strict local optimum.

The OP of Eq. (1) need to be looked at with regard to the following points in order to facilitate a solution to yield the necessary optimality conditions [10]. The points are:

- Make a clear distinction between those inequality constraints, which are "active" (i.e. binding at their limit values) and those, which are "inactive"', i.e., which have a negative functional value in the optimum (satisfied).
- Treat all "active" inequality in the same way as regular equality constraints and determine the optimality conditions for this "pseudo"-equality constrained problem in the same way as for "standard" equality constrained OP.
- In addition, state that all Lagrange-multipliers ($\mu_i$) of all "active" inequality constraints must be positive.
- Also, state that all "inactive" inequality constraints must have a value less than zero (otherwise they would not be "inactive").

We can set up the Lagrangian expression considering all equality and all "active" inequality constraints as follows:

$$\begin{aligned} \min L &= F(\mathrm{x}) + \sum_{i=1}^{m} \lambda_i g_i(\mathrm{x}) + \sum_{j=1}^{p'} \mu_j h_j(\mathrm{x}) \\ &= F(\mathrm{x}) + \lambda^T g(\mathrm{x}) + \mu^T h(\mathrm{x}) \end{aligned} \tag{2}$$

Where $\lambda_i$ and $\mu_i$ are the Lagrange-multipliers. We assume that the first $p'$ inequality constraints are "active" inequality constraints. All the other inequality constraints are "inactive". The optimality conditions for this problem are as follows:

$$\begin{aligned} &\frac{\partial L}{\partial x} = \frac{\partial}{\partial x}(F(\mathrm{x}) + \lambda^T g(\mathrm{x}) + \mu^T h(\mathrm{x})) = 0 \\ &\frac{\partial L}{\partial \lambda} = g(\mathrm{x}) = 0 \\ &\frac{\partial L}{\partial \mu} = h(\mathrm{x}) = 0, \ \mu \ge 0 \\ &h_j(\mathrm{x}) = 0, \ \forall j : \text{"active" inequality constraint} \\ &h_i(\mathrm{x}) < 0, \ \forall i \ : \text{"inactive" inequality constraint} \end{aligned} \tag{3}$$

These points represent the set of necessary optimality conditions for the general non-linear OP of Eq. (1).

*B. Quadratic Programming* (*QP*)

Quadratic Programming is a branch of mathematics that deals with finding extreme values of quadratic functions when the variables are constrained by linear equalities and inequalities [5] [3].

The classic objective function of a QP problem is as follows:

$$\min F = \frac{1}{2}\mathrm{x}^T Q\,\mathrm{x} + c^T\mathrm{x} \tag{4}$$

subject to linearized equality and inequality constraints:

$$\begin{aligned} &A_1\mathrm{x} - b_1 = 0 \\ &A_2\mathrm{x} - b_2 \le 0 \end{aligned} \tag{5}$$

Where, x is the vector of unknowns, dim (x)=$n$; c is the vector of cost coefficients, dim (c)=$n$; $Q$ is an ($n$ . $n$) matrix; $A_1$ is an ($m$ . $n$) matrix; $A_2$ is an ($p$ . $n$) matrix; $b_1$ is the vector specifying the right hand sides of the equality constraints, dim ($b_1$)=$m$; $b_2$ is the vector specifying the right hand sides of the inequality constraints, dim ($b_2$)=$p$

All these matrices and vectors except x are numerically given. In addition, the matrix $Q$ must be positive definite and symmetric. ($Q$ is positive definite if and only if y'.$Q$.y > 0 for all nonzero vectors y). With these conditions for $Q$ the QP describes a convex problem. Note that depending on the OP, the above matrices can be either sparse or compact (i.e. non-sparse/full).

*C. Linear Programming* (*LP*)

Linear programming is a branch of mathematics that deals with finding extreme values of linear functions when the variables are constrained by linear equalities and inequalities [5] [2] [7].

The standard linear programming problem is defined as

$$\min F = c^T x \quad (6)$$

Subject to:

$$A_1 x = b_1$$
$$A_2 x \le b_2 \quad (7)$$
$$x \ge 0 \quad (9)$$

where $x$ is the vector of unknowns, *dim* $[x]=n$; $c$ is the vector of cost coefficients, dim $(c)=n$; $A_1$ is an (m.n) matrix, $m < n$. $A_2$ is an (p.n) matrix ; $b_1$ is the vector specifying the right hand sides of the equality constraints, dim $(b_1)=m$;$b_2$ is the vector specifying the right hand sides of the inequality constraints, dim $(b_2)=p$.

## III. Classification of Optimization Algorithms to Solve General NL-OP

In this section the general OP is solved by an integrated method. All equations and the objective function consist of smooth, twice differentiable function parts. The variables x are continuous, real variables. The goal is to find a solution to this general non-linear OP**.** The nonlinear optimization problem algorithms will be discussed in two classes:

### *A. Class-A: Iterative solution of an approximated LP or QP - OP*

The class-A algorithms are the Methods whereby the optimization starts from a solved Newton-Raphson (NR) process of a well-determined non-linear system of equations [2]. The Jacobian and other sensitivity relations are used in the optimization process, which is usually LP or QP based. The process as a whole is iterative. After each LP or QP iteration the NR is solved again.

#### *1) Successive QP solution of approximated OP*

An approximation around a given operating point $x^0$ leads to the following QP system: A quadratically approximated objective function:

$$\min F = \frac{1}{2}\Delta x^T Q \Delta x + c^T \Delta x \quad (9)$$

subject to linearized equality and inequality constraints:

$$A_1 \Delta x - b_1 = 0$$
$$A_2 \Delta x - b_2 \le 0 \quad (10)$$

where

x is the vector of unknowns, dim(x) $= n$ .

$c = \left(\frac{\partial F}{\partial x}\right)\Big|_{x = x^0}$ is the vector of linearized objective function cost coefficients, dim (c) $= n$ .

$Q = \left(\frac{\partial^2 F}{\partial x^2}\right)\Big|_{x^0}$ is an $(n.n)$ matrix

$A_1 = \frac{\partial g(x)}{\partial x}\Big|_{x = x^0}$ is an $(m.n)$ matrix

$A_2 = \frac{\partial h(x)}{\partial x}\Big|_{x = x^0}$ is an $(p.n)$ matrix

$b_1 = -g(x^0)$ is the vector specifying the right hand sides of the equality constraints, dim $(b_1)=m$

$b_2 = -h(x^0)$ is the vector specifying the right hand sides of the inequality constraints, dim $(b_2)=p$

The iteration loop is exemplified in figure 1.

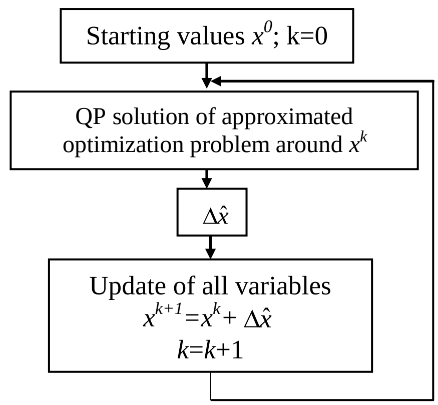


Figure 1. Flow chart of successive QP

#### *2) Successive QP solution of approximated OP with NR support*

The main problem with the iterative procedure of the previous subsection is the choice of the initial solution point $x^0$ which affect the convergence. The general goal is to have a solution point $x^0$ which satisfies the set of equality constraints: $g(x^0) = 0$, but not necessarily the inequality constraints. The problem dimensions have been given before, i.e. dim $(g) = m$, dim $(x) = n$. In addition we assume that $m < n$, i.e. there are fewer equality constraints than variables. Thus a degree of freedom exists for the solution of the equality constraint. The trick is now to split the vector $x$ into two subvectors which allow the solution of a set of equations with the same number of non-linear equations and unknown variables. i.e. the equality constraints can be written as follows: $x^T = [x_1^T, x_2^T]$ and $g(x_1, x_2^0) = 0$, where dim $(x_1) = m$, dim $(x_2) = n-m$.

So the previously discussed form of an iterative QP execution can be extended to an iterative execution of first a NR solution process then a QP algorithm, then a variable update or $\Delta x$, then starting a new NR algorithm with the new computed values for $x_2^0$, then doing again a QP, etc. This iterative procedure is also not much more than a successive execution of a QP, however the QP starts always around a solved set of non-linear equations, see figure 2.

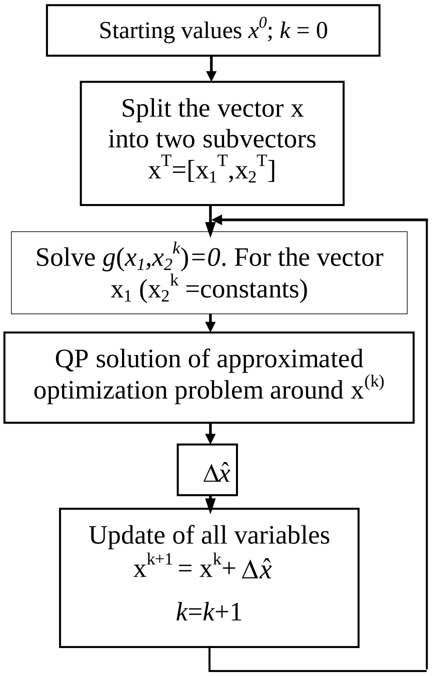


Figure 2. Flow chart of successive QP with NR support

*3) Successive compact QP solution of approximated OP with NR support*

The smaller variable and equality constraint number leads to the term "compact" QP. To get this formulation we eliminate the variables $\Delta x_1$ from Eq. (10). Conceptually the equality constraints in Eq. (10) are split as follows:

$$A_{11}\Delta x_1 + A_{12}\Delta x_2 - b_1 = 0 \qquad (11)$$

From Eq. (11) $\Delta x_1$can be computed as follows:

$$\Delta x_1 = (A_{11})^{-1}(-A_{12}\Delta x_2 + b_1) \qquad (12)$$

Note that the matrix $A_{11}$ must be a square ($m.m$) matrix and it must be non-singular. Then, Eq. (12) exists. This solution for the vector $\Delta x_1$ can be substituted in the original QP formulation of Eq. (9) and Eq. (10), giving the following compact QP:

$$\min F = \frac{1}{2}\Delta x_2^T Q' \Delta x_2 + c'^T \Delta x_2 \qquad (13)$$

Subject to linearized inequality constraints:

$$A'_{22}\,\Delta x_2 - b'_2 \le 0 \qquad (14)$$

with

$$Q' = \left[-A_{12}^T (A_{11}^{-1})^T, U\right] \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} -A_{11}^{-1}A_{12} \\ U \end{bmatrix}$$

$U$ is unity matrix;

$$c_2'^T = c_2^T - c_1^T A_{11}^{-1} A_{12} ;$$

$$A'_{22} = A_{22} - A_{21}A_{11}^{-1}A_{12} ; \qquad (15)$$

$$b'_2 = b_2 - A_{21}A_{11}^{-1}b_1$$

Note that the QP defined by Eq. (13) and Eq. (14) is a compact QP with $n$-$m$ number of variables $\Delta x_2$ and no equality constraints. All equality constraints have been eliminated. The output of this compact QP is $\Delta\hat{x}_2$. Once this vector is obtained one can compute the optimal values for $\Delta\hat{x}_1$ by using Eq. (12). This leads to a new iterative algorithm, shown in figure 3.

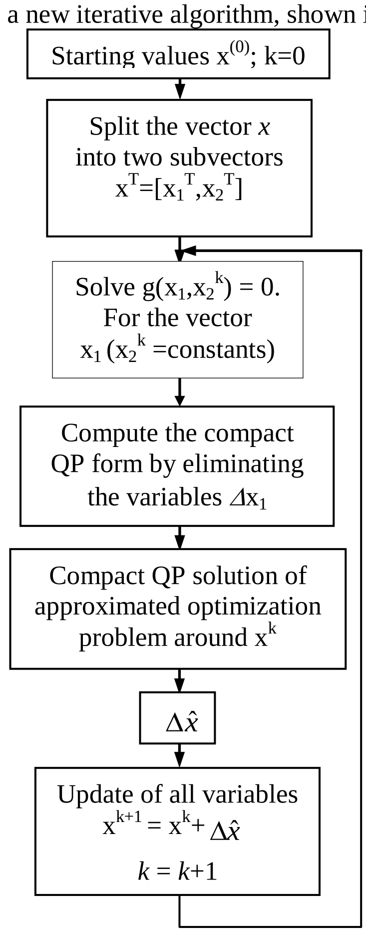


Figure 3. Flow chart of compact QP algorithm.

*B. Class B optimization: Integrated iterative solution of (Kuhn-Tucker) KT-optimality conditions*

The class-B relying on the exact optimality conditions whereby the equality constraints are attached. There is no prior knowledge of the solution of any (sub-) set of equality constraints as done in class-A. The process is iterative and each intermediate solution approaches the optimality conditions.

In this section the optimization formulation is solved by an integrated method as compared to the optimization formulation of the class-A where a set of equations and their solution by a NR is separated from the optimization part [2] [7]. In this work one approach is discussed. It is the so-called non-linear Interior Point (IP) approach [35] which is based on an efficient solution of the non-linear (Kuhn-Tucker) KT optimality conditions using a combination of NR, step-length control and barrier function parameter decrease during all iterations. Other algorithms are found in the literature in this class-B optimization. All class-B algorithms have in common that an iterative solution of possibly transformed non-linear KT optimality conditions will be achieved.

*1) Solution of the KT by Interior Point algorithm (KT optimality conditions)*

For the solution of KT optimality conditions [6] [4] [8], the problem is defined as follows:

$$L = F(x) + \lambda^T g(x) + \mu^T h(x) \tag{16}$$

Note that only variables $x$ are used in the class-B approach. This is slightly different from the class-A approach where a distinction between the control and state variables is advantageous. The optimality conditions can be derived by formulating the Lagrange function: $L$

The first order necessary optimality conditions are as follows:

$$\begin{aligned}
&\frac{\partial L}{\partial x} = \frac{\partial F(x)}{\partial x} + \left(\frac{\partial g(x)}{\partial x}\right)^T \lambda + \left(\frac{\partial h(x)}{\partial x}\right)^T \mu = 0\\
&\frac{\partial L}{\partial \lambda} = g(x) = 0\\
&\mathrm{diag}\{\mu_i\}.\frac{\partial L}{\partial \mu} = \mathrm{diag}\{\mu_i\}.h(x) = 0\\
&\frac{\partial L}{\partial \mu} = h(x) \le 0,\ \mu \ge 0
\end{aligned} \tag{17}$$

The third constraint set together with the last set indicates that an inequality constraint is only active (i.e. being limited) when $\mu_i > 0$, i.e. $h_i(\mathrm{x}) = 0$. For the case where the inequality $i$ is not active at its limit, $h_i(\mathrm{x}) < 0$, $\mu_i = 0$.

*2) Interior Point (IP) optimization*

The idea of the NR for equality constraints is extended to include also the inequality constraints in the formulation. In order to understand the key points of the IP algorithm for a NL-OP [6] [4] [8], we must state the following: The original OP is reformulated as follows [3]:

$$\min F(x) - \zeta \sum_{i=1}^{p} \ln(z_i) \quad (\zeta > 0)$$

Subject to

$$\begin{aligned}
&g(x) = 0\\
&h(x) + z = 0\\
&z > 0
\end{aligned} \tag{18}$$

With the following points taken into consideration:

- The positive barrier parameter $\zeta$ has to become almost zero.
- We have to force the variables $z$ to remain positive during all iterations of the algorithm. This fact gives the algorithm the name "Interior". The term barrier has its justification in that the "barrier function" ($\zeta \sum_{i=1}^{p} \ln(z_i)$) cannot cross the border at zero.

We can formulate now the KT conditions of this new OP assuming implicitly that $z > 0$:

$$L_{IP} = F(x) - \zeta \sum_{i=1}^{p} \ln(z_i) + \lambda^T g(x) + \mu^T (h(x) + z) \tag{19}$$

From this special IP-oriented Lagrangian we derive the KT-conditions:

$$\begin{aligned}
&\frac{\partial L_{IP}}{\partial x} = \frac{\partial F(x)}{\partial x} + \left(\frac{\partial g(x)}{\partial x}\right)^T \lambda + \left(\frac{\partial h(x)}{\partial x}\right)^T \mu = 0\\
&\frac{\partial L_{IP}}{\partial \lambda} = g(x) = 0\\
&\frac{\partial L_{IP}}{\partial z} = \mu - \zeta.\mathrm{diag}\left(\frac{1}{\mathrm{z_i}}\right).e = 0\\
&\frac{\partial L_{IP}}{\partial \mu} = h(x) + z = 0,\ \mu \ge 0,\ z > 0
\end{aligned} \tag{20}$$

Eq. (20) represents IP-KT first order conditions which must be valid at the optimum for any given barrier parameter $\zeta > 0$. The vector $e$ in Eq. (20) indicates a vector with 1's only, dim ($e$)= dim($\mu$) = dim ($z$) = $p$. The principal idea behind the IP solution algorithm of Eq. (20) is as follows:

- Formulate a NR solution step for the equality constraints only. Note, that the number of equality constraints and the number of unknown variables are identical ($n + m + 2\,p$).
- Choose a starting point for the unknown variables in such a way that all variables, which are limited, get a positive value (i.e. $z^0 > 0$, $\mu^0 > 0$). The choice for the starting values is quite sensitive for a convergence success of the algorithm.
- After computing the optimal solution $\Delta x_{\mathrm{opt}}$, $\Delta\lambda_{\mathrm{opt}}$ , $\Delta z_{\mathrm{opt}}$, $\Delta\mu_{\mathrm{opt}}$, of the linear system of one update step with the

NR solution procedure, use a step- length control [11] in such a way that all variables $z$ and $\mu$ remain positive during all iterations of this iterative algorithm. This is done as follows:

- Obviously the barrier parameter $\xi$ must be very near to zero at the optimum. If this is not the case the OP is not the same as originally formulated.

## IV. DESCRIPTIONS OF SOME ENERGY MANAGEMENT SYSTEM OPTIMIZATION FUNCTIONS

Economic Dispatch (ED): This function optimizes the total cost of active power generation, assuming that every generator has a convex cost curve related to its own active power, every generator has upper and lower active power generating limits and it is also assumed that the sum of all active powers of generator must be equal to a given total system load plus total system losses.

Optimal Power Flow (OPF): is an optimization function, which minimizes the total generation cost, the total resistive network or the resistive branch losses for a certain area of the network. At the same time the OPF considers all power flow equations and also operational constraints on the network elements like transmission line current limits and voltage magnitude limits on generator nodes. The OPF has similar goals like the ED, however, it considers the network much more comprehensively than ED.

Unit Commitment: is the optimization function, which determines at what discrete time intervals in the near future (usually hourly intervals) which generators must be ready to deliver power and which one can be shut down. For each time interval the goal is identical with the economic dispatch goal, i.e. the minimization of the total cost of active power generation of all thermal generators. Due to constraints of maximum generator power changes from one time interval to the next this optimization problem is very complex and is a mixture between a discrete and continuous variable optimization.

## V. ECONOMIC DISPATCH PROBLEM SOLUTION BY QUADRATIC PROGRAMMING

Generation allocation is defined as the process of allocating generation levels to the thermal generating units in service within the power system, so that the system load is supplied entirely and most economically [12] and [13]. The objective of the generation allocation or ED problem is to calculate, for a single period of time, the output power of every generating unit so that all demands are satisfied at minimum cost, while satisfying different technical constraints of the network and the generators. The standard ED problem can be described mathematically as an objective with two constraints as:

$$\min\ F_T = \sum_{i=1}^{N} F_i(P_i) \tag{21}$$

Subject to the following constraints:

$$\begin{aligned} &\sum_{i=1}^{N} P_i = D + P_{loss} \\ &P_i^{\min} \le P_i \le P_i^{\max} \end{aligned} \tag{22}$$

where, $N$ is the total number of units in service; $P_i$ is the real power output of i-th generator (MW); $F_T$ is the total operating cost ($ /h); $F_i$ ($P_i$) is the operating cost of unit i ($ /h); $D$ is the total demand (MW); $L$ is the transmission losses (MW); $P_i^{min}$, $P_i^{max}$ are the operating power limits of unit $i$ (MW).

The fuel cost function of a generator that usually used in power system operation and control problem is represented with a second-order polynomial.

$$F_i(P_i) = c_i + b_i P_i + b_i P_i^{\,2} \tag{23}$$

where, $c_i$, $b_i$ and $a_i$ are the cost coefficients (non-negative constants) of the $i$ th generating unit.

We propose the following MATLAB code:

```
x= quadprog (H, f, A, b, Aeq, beq, lb, ub)
% solves the the quadratic programming problem:
min 0.5*x'*H*x + f'*x
% while satisfying the constraints
A*x ≤ b
Aeq*x = beq
lb <= x <= ub
```

To map the ED to QP, the objective function variables are given by the power generation output vector as follow:

$$x = [P_1\ ,\ P_2\ ,...,P_N]^T \tag{24}$$

$$H = 2\times\begin{bmatrix} \dfrac{a_1}{1-2B_{11}P_1 - B_{01}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \dfrac{a_N}{1-2B_{NN}P_{NN} - B_{0N}} \end{bmatrix}^T \tag{25}$$

$$f = \left[\frac{b_1}{1-2B_{11}P_1 - B_{01}},...,\frac{b_N}{1-2B_{NN}P_{NN} - B_{0N}}\right]^T \tag{26}$$

To satisfy the equality constraint Aeq*x = beq, we set:

$$\begin{aligned} Aeq = [1, 1,...,1] + [P_1,\ P_2,...,P_N]\begin{bmatrix} B_{11} & ... & B_{1N} \\ \vdots & \ddots & \vdots \\ B_{N1} & \cdots & B_{NN} \end{bmatrix} + \\ \left[B_{01},\cdots B_{0N}\right] + \left[\frac{B_{00}}{P_1},\frac{B_{00}}{P_2},...,\frac{B_{00}}{P_N}\right] \end{aligned} \tag{27}$$

$$beq\ =\ D + 2P_{loss} \tag{28}$$

Where $P_{loss}$ is power transmission losses calculated by following loss formula commonly known as the B-coefficients formula:

$$P_{loss} = [P_1,\ P_2, ..., P_N] \begin{bmatrix} B_{11} & ... & B_{1N} \\ \vdots & \ddots & \vdots \\ B_{N1} & \cdots & B_{NN} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} + ...$$

$$...\left[B_{01}, \cdots B_{0N}\right] \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} + B_{00} \tag{26}$$

Where, $B_{ij}$, $B_{0i}$ and $B_{00}$ are the loss formula coefficient.

The operating power limits are imposed in the formulation of quadratic programming as follows:

$$lb = [P_1^{\min},\ P_2^{\min}, ..., P_N^{\min}] \tag{27}$$

$$ub = [P_1^{\max},\ P_2^{\max}, ..., P_N^{\max}] \tag{29}$$

To map the ED to QP in MATLAB, we propose the following program:

```
for i=1:10
          Pl=P'*B*P+B01*P+B00;
          Aeq =ones(1,n)+(P'*B+B01+B00/P);
          beq=Pd+2*Pl;
          ll=diag(1-2*B*P-B01');
          A1=inv(ll)*a;
          f=inv(ll)*b;
          H=2*diag(A1);
P=quadprog(H,f,[],[],Aeq,beq,l,u);
pln=P'*B*P+B01*P+B00;
acu=(Pd+pln)-sum(P);
end
```

## VI. Case Study And Results

The IEEE 30 bus system has 6 generating units with the characteristics shown in Table I. The line loses are calculated by the B-coefficients method and given in Table II. The network topology and the test data for the IEEE 30 bus system are given in [14].

TABLE I
THE 6 THERMAL GENERATORS CHARACTERISTICS OF CASE STUDY 1

| Unit N° | $P_{imin}$ | $P_{imax}$ | $a_i$ | $b_i$ | $c_i$ |
|---|---|---|---|---|---|
| 1 | 100 | 500 | 0.007 | 7 | 240 |
| 2 | 50 | 200 | 0.0095 | 10 | 200 |
| 3 | 80 | 300 | 0.009 | 8 | 220 |
| 4 | 50 | 150 | 0.009 | 11 | 200 |
| 5 | 50 | 200 | 0.008 | 10.5 | 220 |
| 6 | 50 | 120 | 0.0075 | 12 | 190 |

TABLE II
B- COEFFICIENTS OF IEEE 30-BUS 6-UNIT SYSTEM

$$B = 10^{-4} \begin{bmatrix} 2.231 & 1.162 & -0.122 & -0.017 & 0.113 & 0.39 \\ 1.162 & 1.89 & -0.077 & -0.048 & 0.069 & 0.28 \\ -0.122 & -0.077 & 2.004 & -0.74 & -0.724 & -0.599 \\ -0.017 & -0.048 & -0.74 & -1.479 & 0.538 & 0.342 \\ 0.113 & 0.069 & -0.724 & 0.538 & 1.185 & 0.053 \\ 0.39 & 0.28 & -0.599 & 0.342 & 0.053 & 2.34 \end{bmatrix}$$

$$B_0 = 10^{-5}\left[0.38 \quad 1.79 \quad -5.32 \quad 1.52 \quad 2.33 \quad 1.26\right]$$

$$B_{00} = 0.00154$$

We have compared the developed algorithm to other economic dispatch algorithm, Table III show the comparison between QP algorithm and $\lambda$ iteration algorithm [11] for 8 Times intervals.

TABLE III
THE TOTAL COST POWER GENERATION OF 8 TIME PERIOD FOR THE THERMAL UNITS WITH LOSSES AND GENERATION LIMIT AND WITHOUT RAMP-RATE LIMIT

| Hour (h) | Load (MW) | Total Cost with QP ($/h) | Total Cost with λ iteration ($/h) | saving ($/h) |
|---|---|---|---|---|
| 1 | 955 | 11797.8396 | 11839.803 | 41.963 |
| 4 | 930 | 11464.9621 | 11505.290 | 40.327 |
| 7 | 989 | 12253.9174 | 12298.848 | 44.930 |
| 10 | 1150 | 14478.1677 | 14538.501 | 60.333 |
| 13 | 1190 | 15049.3433 | 15117.104 | 67.760 |
| 16 | 1250 | 15946.8412 | 16025.133 | 78.291 |
| 19 | 1159 | 14605.7259 | 14667.566 | 61.840 |
| 22 | 984 | 12186.569 | 12231.043 | 44.473 |

The results of the economic dispatch for the 6-units test system are listed in Table III, and it show the performance of the proposed QP method with a valuable ($/h) saving comparing to $\lambda$ iteration method. The execution time of the adapted QP algorithm for economic dispatch is faster than the lambda method where the computational time is about 0.2 second on a Pentium IV, 3 GHz.

## VII. Conclusion

Optimization methods are judged by their performance with respect to speed, versatility and robustness. There is no single optimization method which meets all requirements satisfactorily and which can be classified as the best non-linear optimization problem solution algorithm. Class-A and class-B methods have their relative merits and perform well for one or the other particular application. In any one problem, however, a method could show poor performance. In both class-A and class-B algorithms the size of the linear inequality constraint set is identical and thus a distinction between both methods does not exist with respect to this point. Class-B methods are also attractive. They solve all objective function problems with no particular differences during the solution process which is a strong benefit for the class-B methods. This comes mainly from the fact that the class-B algorithms solve the optimality conditions of the original OP directly, where as the class-A algorithms solve only optimality conditions of the approximated OP. The disadvantages of the class-B algorithms lie in the fact that the number of variables to be handled iteratively is quite large (much larger than in the class-A algorithms where the number of variables is significantly reduced). Generally it is not clear which one of the classes is better. In the end the decision may be made based on the best combination of algorithmic robustness, computer code efficiency and computer code maintainability [2][8].

This paper presents a QP formulation for ED taking into consideration the generation limits, transmission losses. The demand is assumed to be periodic. We applied the QP approach to the periodic implementation of the optimal solutions of ED problem problems. The convergence and robustness of the QP algorithms are demonstrated through the application of QP to a 6-unit IEEE test system.

The results showed that the differences in Total cost results between the QP approach and λ iteration method are satisfactory, which checks the validity of this study.

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