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A note on LQP method for nonlinear complimentarity problems

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Abstract. Inspired and motivated by the results of Bnouhachem et al. [5], we propose and analyze a new LQP method for solving nonlinear complementarity problems by performing an additional projection step at each iteration and another optimal step length is employed to reach substantial progress in each iteration. Under certain conditions, we show that the proposed method is globally convergent. We report some preliminary computational results to illustrate the efficiency of the proposed method.

Key words. Nonlinear complementarity problems, monotone operators, logarithmic-quadratic proximal methods.

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1 Introduction

The nonlinear complementarity problem (NCP) is to determine a vector $x \in R^n$ such that

$$x \geq 0, \quad F(x) \geq 0 \quad \text{and} \qquad x^T F(x) = 0, \tag{1.1}$$

where F is a nonlinear mapping from R^n into itself. Throughout this paper we assume that F is continuously differentiable and monotone with respect to R^n_+, and the solution set of (1.1), denoted by Ω^*, is nonempty.

Complementarity problems introduced by Lemke [17] and Cottle Dantzig [8] in the early 1960's. These problems are being used as a powerful tool to study a wide class of problems with applications in industry, engineering, optimization, mathematical and physical sciences in a unified framework. It have been shown that the linear and nonlinear problems in operations can be formulated as complementarity problems, which can be solved more effectively. There are several methods for solving the complementarity problems, which can be divided into two categories namely direct and indirect(iterative) methods. Direct methods are those based on the process of pivoting, which are mainly due to Lemke [17] and Cottle and Dantzig [8]. The practicality of the direct methods is restricted mainly due to the problem size limitations in computer implementations. Also these methods can not be extended for nonlinear complementarity problems. These facts and reasons have stimulated much investigation of alternative approaches for solving the nonlinear complementarity problems. In this paper, we are only concerned with the iterative methods of proximal point methods. These iterative methods have emerged in the last decades as a powerful technique for solving the nonlinear complementarity problems effectively. These methods are user friendly and can be implemented easily. It is well-known that the complementarity problems can be formulated as a variational inclusion involving the sum of the two monotone operator. This equivalent formulation has played an important part in suggesting and developing proximal point algorithms for solving the complementarity problems. Rockafellar [22, 23] gave the approximate proximal point algorithm, which is more practical and attractive than the exact one. Given $\beta_k \geq \beta > 0$, the inexact version of the proximal point algorithm generates iteratively sequence $\{x^k\}$ satisfying

$$\xi^k \in \beta_k T(x) + \nabla_x q(x, x^k), \tag{1.2}$$

where $\xi^k \in \mathcal{R}^n$ is the error term,

$$q(x, x^k) = \frac{1}{2}\|x - x^k\|^2 \tag{1.3}$$

and $T(x) = F(x) + N_{R^n_+}(x)$, where $N_{R^n_+}(.)$ is the normal cone operator to R^n_+ defined by

$$N_{R^n_+}(x) := \begin{cases} \{y : y^T(v - x) \leq 0, \quad \forall v \in R^n_+\}, & \text{if} \quad x \in R^n_+; \\ \emptyset, & \text{otherwise.} \end{cases}$$

Proposition 1.1[10, 21] Let $K \subset R^n$ be nonempty closed and convex and $F : K \longrightarrow R^n$ continuous. The following properties hold:

(a) N_K is maximal monotone.

(b) If F is monotone then $T(x) = F(x) + N_K(x)$ is maximal monotone.

(c) $N_K(x) = \{0\}$ when $K = R^n$ or when $x \in \text{int}K$, the interior of K.

Several works [6, 9, 14, 16, 24, 26] have been concentrated on the generalization of the proximal algorithm replacing the usual quadratic term by some nonlinear functionals $r(x, x^k)$. Auslender et al. [2, 3] proposed a new type of proximal interior method through replacing the quadratic function (1.3) by $d_\phi(x, x^k)$ which could be defined as

$$d_\phi(x, y) = \sum_{j=1}^{n} y_j^2 \phi(y_j^{-1} x_j)$$

where

$$\phi(t) = \begin{cases} \frac{1}{2}(t-1)^2 + \mu(t - \log t - 1), & \text{if} \quad t > 0; \\ +\infty, & \text{otherwise.} \end{cases}$$

and $\mu \in (0, 1)$. Then the problem (1.2) becomes for given $x^k \in R^n_{++}$ and $\beta_k \geq \beta > 0$, the new iterate x^{k+1} is solution of the following set-valued equation:

$$\xi^k \in \beta_k T(x) + \nabla_x Q(x, x^k), \tag{1.4}$$

where

$$Q(x, x^k) = \begin{cases} \frac{1}{2}\|x - x^k\|^2 + \mu \sum\limits_{j=1}^{n} \Big((x_j^k)^2 \log \frac{x_j^k}{x_j} + x_j x_j^k - (x_j^k)^2 \Big), & \text{if} \quad x \in R^n_{++}; \\ +\infty, & \text{otherwise.} \end{cases} \tag{1.5}$$

Auslender et al. [2] proved that the sequence $\{x^k\}$ generated by (1.2) converges under the following conditions:

$$\sum_{k=1}^{+\infty} \|\xi^k\| < +\infty \quad \text{and} \quad \sum_{k=1}^{+\infty} \langle \xi^k, x^k \rangle \text{ exists and is finite.} \tag{1.6}$$

Note that (1.6) implies that (1.2) should be solved exactly. To release this difficulty Burachik and Svaiter [7] presented a meaningful modification of the inexact LQP method with attractive characteristic that the relative error $\frac{\|\xi^k\|}{\|x^k - x^{k+1}\|}$ can be fixed on a constant.

It is easy to see that

$$\nabla_x Q(x, x^k) = x - (1-\mu)x^k - \mu X_k^2 x^{-1},$$

where $X_k = \text{diag}(x_1^k, x_2^k, \ldots, x_n^k)$, x^{-1} is an n-vector whose j-th element is $1/x_j$ and $\mu \in (0,1)$ is a given constant. From Proposition 1.1(c), we have $T(x) = F(x)$ then the problem (1.4)-(1.5) is equivalent to the following systems of nonlinear equations

$$\beta_k F(x) + x - (1-\mu)x^k - \mu X_k^2 x^{-1} = \xi^k. \tag{1.7}$$

Recently, He et al. [15] , Xu et al. [27] and Bnouhachem et al. [5] introduced some LQP based prediction-correction methods and make the LQP method more practical. Each iteration of the the above methods contains a prediction and a correction, the predictor is obtained via solving the nonlinear equation system (1.7) under significantly relaxed accuracy criterion and the new iterate is computed directly by an explicit formula derived from the original LQP method for [15], while the new iterate is computed by using the projection operator for [27] and [5]. Inspired and motivated by the above research, we propose and analyze a new LQP method for solving nonlinear complementarity problems (1.1) by performing an additional projection step at each iteration and another optimal step length is employed to reach substantial progress in each iteration, which provides a significant refinement and improvement of the methods in [27] and in [5]. Some preliminary computational results are given to illustrate the efficiency of the proposed method.

2 Preliminaries

We list some important results which will be required in our following analysis.

First, we denote $P_{R_+^n}(.)$ as the projection under the Euclidean norm, i.e.,

$$P_{R_+^n}(z) = \min\{\|z - x\|| \quad x \in R_+^n\}.$$

A basic property of the mapping of projection is

$$(y - P_{R_+^n}(y))^T(P_{R_+^n}(y) - x) \geq 0, \quad \forall y \in R^n, \quad \forall x \in R_+^n. \tag{2.1}$$

From (2.1), it is easy to verify that

$$\|P_{R_+^n}(v) - u\|^2 \leq \|v - u\|^2 - \|v - P_{R_+^n}(v)\|^2, \qquad \forall v \in R^n, u \in R_+^n. \tag{2.2}$$

Definition 2.1 The operator $F : R^n \to R^n$ is said to be monotone, if

$$\forall u, v \in R^n, \qquad (v - u)^T(F(v) - F(u)) \geq 0.$$

The following lemma is similar to Lemma 2 in [2]. Hence the proof will be omitted.

Lemma 2.1 [15, 27] For given $x^k > 0$ and $q \in R^n$, let x be the positive solution of the following equation:

$$q + x - (1 - \mu)x^k - \mu X_k^2 x^{-1} = 0, \tag{2.3}$$

then for any $y \geq 0$ we have

$$\langle y - x, q\rangle \geq \tfrac{1+\mu}{2}(\|x - y\|^2 - \|x^k - y\|^2) + \tfrac{1-\mu}{2}\|x^k - x\|^2. \tag{2.4}$$

3 The proposed method

In this section, we propose and analyze the new modified LQP method for solving nonlinear complementarity problems (1.1). For $\sigma \in (0, 1), m_1 \geq 1$ and $m_2 \geq 2$, for given $x^k > 0$ and $\beta_k > 0$, each iteration of the proposed method consists of three steps, the first step offers $\tilde{x}^k$, the second step makes $\bar{x}^k$ and the third step produces the new iterate x^{k+1}.

First step: Find an approximate solution $\tilde{x}^k$ of (1.7), such that

$$0 \approx \beta_k F(\tilde{x}^k) + \tilde{x}^k - (1 - \mu)x^k - \mu X_k^2(\tilde{x}^k)^{-1} = \xi^k \tag{3.1}$$

and ξ^k satisfies

$$\|\xi^k\| \leq \eta\|x^k - \tilde{x}^k\|, \qquad 0 < \mu, \eta < 1. \tag{3.2}$$

Second step: $\bar{x}^k(\alpha_k)$ is defined by

$$\bar{x}^k(\alpha_k) = P_{R^n_+}\Big[x^k - \frac{\alpha_k\beta_k}{1+\mu}F(\tilde{x}^k)\Big], \tag{3.3}$$

where

$$\alpha_k = \max_{\alpha}\{\alpha^*_{k_1} \le \alpha \le m_2\alpha^*_{k_1} \mid \Psi(\alpha) \ge \sigma\Psi(\alpha^*_{k_1})\}, \tag{3.4}$$

$$\alpha^*_{k_1} = \arg\max_{\alpha}\{\Psi(\alpha) \mid \quad 0 < \alpha \le m_1\alpha^*_{k_2}\}. \qquad \Psi(\alpha) \text{ will be defined by (3.15)} \tag{3.5}$$

and

$$\alpha^*_{k_2} = \arg\max_{\alpha}\{\Phi(\alpha) | \quad \alpha > 0\}. \qquad \Phi(\alpha) \text{ will be defined by (3.16)} \tag{3.6}$$

Third step: For $\tau > 0$ and $0 < \rho < 1$, the new iterate $x^{k+1}(\tau)$ is defined by

$$x^{k+1}(\tau) = \rho x^k + (1-\rho)P_{R^n_+}\Big[x^k - \tau(x^k - \bar{x}^k(\alpha_k))\Big]. \tag{3.7}$$

How to choose values of τ to ensure that $x^{k+1}(\tau)$ is closer to the solution set than x^k. For this purpose, we define

$$\Upsilon(\tau) = \|x^k - x^*\|^2 - \|x^{k+1}(\tau) - x^*\|^2. \tag{3.8}$$

Theorem 3.1 Let $x^* \in \Omega^*$, then we have

$$\Upsilon(\tau) \ge (1-\rho)\left(\tau\{\|x^k - \bar{x}^k(\alpha_k)\|^2 + \Theta(\alpha_k)\} - \tau^2\|x^k - \bar{x}^k(\alpha_k)\|^2\right), \tag{3.9}$$

where

$$\Theta(\alpha_k) = \|x^k - x^*\|^2 - \|\bar{x}^k(\alpha_k) - x^*\|^2. \tag{3.10}$$

Proof: Since $x^* \in \Omega^* \subset R^n_+$ and let $x^k_*(\tau) = P_{R^n_+}\Big[x^k - \tau(x^k - \bar{x}^k(\alpha_k))\Big]$ it follows from (2.2) that

$$\|x^k_*(\tau) - x^*\|^2 \le \|x^k - \tau(x^k - \bar{x}^k(\alpha_k)) - x^*\|^2 - \|x^k - \tau(x^k - \bar{x}^k(\alpha_k)) - x^k_*(\tau)\|^2. \tag{3.11}$$

On the other hand, we have

$$\begin{aligned}\|x^{k+1}(\tau) - x^*\|^2 &= \|\rho(x^k - x^*) + (1-\rho)(x^k_*(\tau) - x^*)\|^2\\ &= \rho^2\|x^k - x^*\|^2 + (1-\rho)^2\|x^k_*(\tau) - x^*\|^2 + 2\rho(1-\rho)(x^k - x^*)^T(x^k_*(\tau) - x^*).\end{aligned}$$

Using the following identity

$$2(a+b)^T b = \|a+b\|^2 - \|a\|^2 + \|b\|^2$$

for $a = x^k - x_*^k(\tau), b = x_*^k(\tau) - x^*$ and (3.11), and using $0 < \rho < 1$, we obtain

$$\begin{aligned}
\|x^{k+1}(\tau) - x^*\|^2 &= \rho^2\|x^k - x^*\|^2 + (1-\rho)^2\|x_*^k(\tau) - x^*\|^2 + \rho(1-\rho)\{\|x^k - x^*\|^2 \\
&\quad -\|x^k - x_*^k(\tau)\|^2 + \|x_*^k(\tau) - x^*\|^2\} \\
&= \rho\|x^k - x^*\|^2 + (1-\rho)\|x_*^k(\tau) - x^*\|^2 - \rho(1-\rho)\|x^k - x_*^k(\tau)\|^2 \\
&\le \rho\|x^k - x^*\|^2 + (1-\rho)\|x^k - \tau(x^k - \bar{x}^k(\alpha_k)) - x^*\|^2 \\
&\quad -(1-\rho)\|x^k - \tau(x^k - \bar{x}^k(\alpha_k)) - x_*^k(\tau)\|^2 - \rho(1-\rho)\|x^k - x_*^k(\tau)\|^2. \\
&\le \|x^k - x^*\|^2 - (1-\rho)\{\|x^k - x_*^k(\tau) - \tau(x^k - \bar{x}^k(\alpha_k))\|^2 + \rho\|x^k - x_*^k(\tau)\|^2 \\
&\quad -\tau^2\|x^k - \bar{x}^k(\alpha_k)\|^2 + 2\tau(x^k - x^*)^T(x^k - \bar{x}^k(\alpha_k))\} \\
&\le \|x^k - x^*\|^2 - (1-\rho)\{2\tau(x^k - x^*)^T(x^k - \bar{x}^k(\alpha_k)) - \tau^2\|x^k - \bar{x}^k(\alpha_k)\|^2\}.
\end{aligned}$$

Using the definition of $\Upsilon(\tau)$, we get

$$\begin{aligned}
\Upsilon(\tau) &\ge (1-\rho)\{2\tau(x^k - x^*)^T(x^k - \bar{x}^k(\alpha_k)) - \tau^2\|x^k - \bar{x}^k(\alpha_k)\|^2\} \\
&= (1-\rho)(2\tau\{\|x^k - \bar{x}^k(\alpha_k)\|^2 - (x^* - \bar{x}^k(\alpha_k))^T(x^k - \bar{x}^k(\alpha_k))\} \\
&\quad -\tau^2\|x^k - \bar{x}^k(\alpha_k)\|^2).
\end{aligned} \tag{3.12}$$

Using the following identity

$$(x^* - \bar{x}^k(\alpha_k))^T(x^k - \bar{x}^k(\alpha_k)) = \frac{1}{2}\left(\|\bar{x}^k(\alpha_k) - x^*\|^2 - \|x^k - x^*\|^2\right) + \frac{1}{2}\|x^k - \bar{x}^k(\alpha_k)\|^2,$$

implies

$$\|x^k - \bar{x}^k(\alpha_k)\|^2 - 2(x^* - \bar{x}^k(\alpha_k))^T(x^k - \bar{x}^k(\alpha_k)) = \|x^k - x^*\|^2 - \|\bar{x}^k(\alpha_k) - x^*\|^2. \tag{3.13}$$

Substituting (3.13) in (3.12) and using the notation of $\Theta(\alpha_k)$, we get the assertion of this theorem. □

The following results will be used in the consequent analysis.

Theorem 3.2 [5] Let $x^* \in \Omega^*$ and $\Theta(\alpha_k)$ be defined by (3.10) respectively, then we have

$$\Theta(\alpha_k) \ge \Psi(\alpha_k) \ge \Phi(\alpha_k) \tag{3.14}$$

where

$$\Psi(\alpha_k) = \|x^k - \bar{x}^k(\alpha_k)\|^2 + \frac{2\alpha_k \beta_k}{1+\mu}(\bar{x}^k(\alpha_k) - \tilde{x}^k)^T F(\tilde{x}^k). \tag{3.15}$$

$$\Phi(\alpha_k) = 2\alpha_k \varphi_k - \alpha_k^2 \|d^k\|^2. \tag{3.16}$$

$$\varphi_k = \frac{1}{1+\mu}\|x^k - \tilde{x}^k\|^2 + \frac{1}{1+\mu}(x^k - \tilde{x}^k)^T \xi^k \tag{3.17}$$

and

$$d^k = (x^k - \tilde{x}^k) + \frac{1}{1+\mu}\xi^k. \tag{3.18}$$

Note that $\Phi(\alpha)$ is a quadratic function of α and it reaches its maximum at

$$\alpha_{k_2}^* = \frac{\varphi_k}{\|d^k\|^2} \tag{3.19}$$

and

$$\Phi(\alpha_{k_2}^*) = \alpha_{k_2}^* \varphi_k. \tag{3.20}$$

The next lemma shows that $\alpha_{k_2}^*$ is bounded away from zero.

Lemma 3.1 [5] For given $x^k \in R_{++}^n$ and $\beta_k > 0$, let $\tilde{x}^k$ and ξ^k satisfy the condition (3.2), then we have the following

$$\alpha_{k_2}^* \geq \frac{1-\eta}{2(1+\mu)} \tag{3.21}$$

and

$$\varphi_k \geq \left(\frac{1-\eta}{1+\mu}\right)\|x^k - \tilde{x}^k\|^2. \tag{3.22}$$

Proposition 3.1 [5] Let $\alpha_{k_1}^*$ and $\alpha_{k_2}^*$ be defined by (3.5) and (3.6) respectively, F be monotone and continuously differentiable, then we have

(i) $\|x^k - x^*\|^2 - \|\bar{x}^k(\alpha_{k_1}^*) - x^*\|^2 \geq \Psi(\alpha_{k_1}^*)$

(ii) $\|x^k - x^*\|^2 - \|\bar{x}^k(\alpha_{k_2}^*) - x^*\|^2 \geq \Phi(\alpha_{k_2}^*)$

(iii) $\Psi(\alpha_{k_1}^*) \geq \Phi(\alpha_{k_2}^*)$

Furthermore, if $\Psi'(\alpha_{k_1}^*) = 0$, we have

(iv) $\|x^k - x^*\|^2 - \|\bar{x}^k(\alpha_{k_1}^*) - x^*\|^2 \geq \|x^k - \bar{x}^k(\alpha_{k_1}^*)\|^2.$

4 Convergence analysis

In this section, we consider the convergence analysis of the proposed method.

By using Theorem 3.1 and Theorem 3.2, we get

$$\Upsilon(\tau) \geq (1-\rho)\Lambda(\tau), \tag{4.1}$$

where

$$\Lambda(\tau) = \tau\{\|x^k - \bar{x}^k(\alpha_k)\|^2 + \Psi(\alpha_k)\} - \tau^2\|x^k - \bar{x}^k(\alpha_k)\|^2. \tag{4.2}$$

$\Lambda(\tau_k)$ measures the progress obtained in the k-th iteration. It is natural to choose a step length τ_k which maximizes the progress. Note that $\Lambda(\tau_k)$ is a quadratic function of τ_k and it reaches its maximum at

$$\tau_k^* = \frac{\|x^k - \bar{x}^k(\alpha_k)\|^2 + \Psi(\alpha_k)}{2\|x^k - \bar{x}^k(\alpha_k)\|^2}$$

and

$$\Lambda(\tau_k^*) = \frac{\tau_k^*\{\|x^k - \bar{x}^k(\alpha_k)\|^2 + \Psi(\alpha_k)\}}{2}. \tag{4.3}$$

Remark 4.1 Note that if $\tau_k^* = 1$ the proposed method reduces to the method in [5] Since τ_k^* is to maximize the profit function $\Lambda(\tau)$, we have

$$\Lambda(\tau_k^*) \geq \Lambda(1). \tag{4.4}$$

Inequalities (4.1) and (4.4) show theoretically that the proposed method with $\tau_k^* \neq 1$ is expected to make more progress than that in [5].

Recall that

$$\Psi(\alpha_k) \geq \sigma\Psi(\alpha_{k_1}^*) \geq \sigma\Phi(\alpha_{k_2}^*),$$

using (3.20), (3.21) and (3.22), we have

$$\begin{aligned} \Psi(\alpha_k) \geq \sigma\Phi(\alpha_{k_2}^*) &= \sigma\alpha_{k_2}^*\varphi_k \\ &\geq \frac{\sigma(1-\eta)^2}{2(1+\mu)^2}\|x^k - \tilde{x}^k\|^2. \end{aligned} \tag{4.5}$$

Since $\Psi(\alpha_k) > 0$, and from the definition of τ_k^* it is easy to prove that

$$\tau_k^* \geq \frac{1}{2} \tag{4.6}$$

and

$$\begin{aligned}\Lambda(\tau_k^*) &\geq \Lambda(1)\\ &= \Psi(\alpha_k)\\ &\geq \frac{\sigma(1-\eta)^2}{2(1+\mu)^2}\|x^k-\tilde{x}^k\|^2. \end{aligned} \tag{4.7}$$

For fast convergence, we take a relaxation factor $\gamma \in [1,2)$ and the step-size τ_k by $\tau_k = \gamma\tau_k^*$. Through simple manipulations, we obtain

$$\begin{aligned}\Lambda(\gamma\tau_k^*) &= \gamma\tau_k^*\{\|x^k-\bar{x}^k(\alpha_k)\|^2+\Psi(\alpha_k)\}-(\gamma^2\tau_k^*)(\tau_k^*\|x^k-\bar{x}^k(\alpha_k)\|^2)\\ &= \gamma(2-\gamma)\Lambda(\tau_k^*) \end{aligned} \tag{4.8}$$

It follows from (3.8), (4.1), (4.7) and (4.8) that there is a constant $c>0$ such that

$$\|x^{k+1}-x^*\|^2 \leq \|x^k-x^*\|^2 - c\|x^k-\tilde{x}^k\|^2 \qquad \forall x^* \in \Omega^*. \tag{4.9}$$

The following result can be proved by similar arguments as those in [4, 15, 27]. Hence the proof will be omitted.

Theorem 4.1 [4, 15, 27] If $\inf_{k=0}^{\infty}\beta_k = \beta > 0$, then the sequence $\{x^k\}$ generated by the proposed method converges to some x^∞ which is a solution of NCP.

5 Preliminary Computational Results

Note that in the special case $\xi^k = \beta_k(F(\tilde{x}^k)-F(x^k))$, (3.1) can be written as

$$\beta_k F(x^k) + \tilde{x}^k - (1-\mu)x^k - \mu X_k^2(\tilde{x}^k)^{-1} = 0, \tag{5.1}$$

the solution of (5.1) can be componentwise obtained by

$$\tilde{x}_j^k = \frac{(1-\mu)x_j^k - \beta_k F_j(x^k) + \sqrt{[(1-\mu)x_j^k - \beta_k F_j(x^k)]^2 + 4\mu(x_j^k)^2}}{2}. \tag{5.2}$$

Moreover for any $x^k > 0$ we have always $\tilde{x}^k > 0$.

We now describe the new algorithm as follows.

Step 0. Let $\beta_0 > 0$, $\varepsilon > 0$, $0 < \mu < 1$, $0 < \sigma < 1$, $0 < \eta < 1$, $0 < \rho < 1$, $m_1 \geq 1$, $m_2 \geq 2$, $1 \leq \gamma < 2, x^0 > 0$ and set $k := 0$.

Step 1. If $\|\min(x^k, F(x^k))\|_\infty \le \epsilon$, then stop. Otherwise, go to Step 2.

Step 2. $s := (1-\mu)x^k - \beta_k F(x^k)$, $\qquad \tilde{x}_i^k := \left(s_i + \sqrt{(s_i)^2 + 4\mu(x_i^k)^2}\,\right)/2,$

$\xi^k := \beta_k(F(\tilde{x}^k) - F(x^k))$, $\qquad r := \|\xi^k\|/\|x^k - \tilde{x}^k\|.$

while $(r > \eta)$

$\beta_k := \beta_k * 0.8/r,$

$s := (1-\mu)x^k - \beta_k F(x^k)$, $\qquad \tilde{x}_i^k := \left(s_i + \sqrt{(s_i)^2 + 4\mu(x_i^k)^2}\,\right)/2,$

$\xi^k := \beta_k(F(\tilde{x}^k) - F(x^k))$, $\qquad r := \|\xi^k\|/\|x^k - \tilde{x}^k\|.$

end while

Step 3. Searching step size α_k^* :

Let $\bar{\alpha}_k = \arg\max_\alpha\{\Phi(\alpha) \mid \alpha > 0\}$, where $\Phi(\alpha)$ is defined by (3.16).

Solve the following optimization problem

$\alpha_k^* = \arg\max_\alpha\{\Psi(\alpha) \mid 0 < \alpha \le m_1\bar{\alpha}_k\}$, where $\Psi(\alpha)$ is defined by (3.15).

Step 4. Extending the step size:

$\alpha_k = \max_\alpha\{\alpha_k^* \le \alpha \le m_2\alpha_k^* \mid \Psi(\alpha) \ge \sigma\Psi(\alpha_k^*)\}$

$\bar{x}^k = P_{R_+^n}[x^k - \frac{\alpha_k\beta_k}{1+\mu}F(\tilde{x}^k)],$

$\tau_k^* = \frac{\|x^k - \bar{x}^k(\alpha_k)\|^2 + \Psi(\alpha_k)}{2\|x^k - \bar{x}^k(\alpha_k)\|^2}, \qquad \tau_k = \gamma\tau_k^*,$

and the new iterate is defined by

$x^{k+1} = \rho x^k + (1-\rho)P_{R_+^n}\left[x^k - \tau_k(x^k - \bar{x}^k(\alpha_k))\right].$

Step 5. $\beta_{k+1} = \begin{cases} \frac{\beta_k * 0.7}{r} & \text{if} \quad r \le 0.5; \\ \beta_k & \text{otherwise.} \end{cases}$

Step 6. $k := k+1$; go to Step 1.

To test the proposed method, we consider the nonlinear complementarity problems:

$$x \ge 0, \qquad F(x) \ge 0, \qquad x^T F(x) = 0, \tag{5.3}$$

where

$$F(x) = D(x) + Mx + q,$$

$D(x)$ and $Mx + q$ are the nonlinear part and linear part of $F(x)$ respectively.

We form the linear part in the test problems similarly as in Harker and Pang [13]. The matrix $M = A^T A + B$, where A is an $n \times n$ matrix whose entries are randomly generated in the interval $(-5, +5)$ and a skew-symmetric matrix B is generated in the same way. The vector q is generated from a uniform distribution in the interval $(-500, 500)$. In $D(x)$, the nonlinear part of $F(x)$, the components are chosen to be $D_j(x) = d_j * \arctan(x_j)$, where d_j is a random variable in $(0, 1)$. A similar type of problems was tested in [18] and [25].

In all tests we take $\rho = 0.1, \sigma = 0.05, m_1 = 3, m_2 = 4, \eta = 0.9, \gamma = 1.98$ and the logarithmic proximal parameter $\mu = 0.1$. All iterations start with $x^0 = (1, ..., 1)^T$ and $\beta_0 = 1$, and stopped whenever

$$\| \min(x^k, F(x^k)) \|_\infty \leq 10^{-7}.$$

All codes were written in Matlab, we compare the proposed method with those in [5] and [27], the test results for problem (5.3) are reported in Table 5.1. k is the number of iterations and l denotes the number of evaluations of mapping F.

The numerical results show that the proposed method is very efficient algorithm even for large-scale classical NCP. Moreover, the new step size τ_k plays important role to reduce the iterative numbers and the evaluation numbers of F. Moreover, it demonstrates computationally that the new method is more effective than the methods presented in [5] and [27] in the sense that the new method needs fewer iteration and less evaluation numbers of F, which clearly illustrate its efficiency and thus justifies the theoretical assertions.

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Table 5.1 Numerical results for problem (5.3)

Dimension of the problem	The method in [27]		The method in [5]		The proposed method	
	k	l	k	l	k	l
n=200	500	1073	408	849	257	551
n=300	557	1193	442	913	287	604
n=500	618	1324	490	1013	318	677
n=700	600	1278	474	989	303	644
n=1000	585	1244	455	947	295	568

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