An SQP Algorithm with Equality Constrained Subproblems for Nonlinear Inequality Constrained Optimization[1]

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Abstract. In this paper, an improved SQP method is proposed to solve the nonlinear programming problem, the direction d_0^k is only necessary to solve a equality constrained quadratic programming, the feasible direction with descent d^k and the high-order revised direction $\widetilde{d^k}$ which avoids Maratos effect are obtained by explicit formulas. Furthermore, the global and superlinear convergence are proved under some suitable conditions.

Key words. Nonlinear inequality, Constrained optimization, SQP method, Equality constrained quadratical programming, Global convergence, Superlinear convergence rate

1. Introduction

Consider the nonlinear inequality constrained optimization problem:

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & g_j(x) \leq 0, j \in I = \{1, 2, \ldots, m\}, \end{array} \tag{1.1}$$

where $f, g_j : R^n \to R (j \in I)$ are continuously differentiable functions. Denote the feasible set for (1.1) by $X = \left\{x \in R^n \mid g_j(x) \leq 0, j \in I\right\}$.

A point $x \in X$ is said to be a KKT point of (1.1), if it is feasible and satisfies the equalities

$$\begin{array}{l} \nabla f(x) + \sum\limits_{j=1}^{m} \lambda_j \nabla g_j(x) = 0, \\ \lambda_j g_j(x) = 0, j \in I, \end{array} \tag{1.2}$$

where $\lambda = (\lambda_1, \ldots, \lambda_m)^T$ is nonnegative, and λ is said to be the corresponding KKT multiplier vector.

Due to superlinear convergence rate, the SQP (sequential quadratic programming) methods are currently considered one of the most effective methods for solving nonlinearly constrained optimization problems [1] - [8]. SQP algorithms generate iteratively the main search direction d_0 by solving the following quadratic programming subproblem:

$$\begin{array}{ll} \min & \nabla f(x)^T d + \frac{1}{2} d^T H d \\ \text{s.t.} & g_j(x) + \nabla g_j(x)^T d \leq 0, j = 1, 2, \ldots, m, \end{array} \tag{1.3}$$

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where $H \in R^{n\times n}$ is a symmetric positive definite matrix. However, such type SQP algorithms have two serious shortcomings: (1) SQP algorithms require that the relate QP subproblem (1.3) must be consistency. (2) There exists Matatos effect [6]. Many efforts have been made to overcome the shortcomings through modifying the quadratic subproblem (1.3) and the direction d [5]-[13].

P. Spellucci.[12] proposed a new SQP algorithm for solving general nonlinear programs. For the problem (1.1), the d_0 is obtained by solving QP subproblem with only equality constraints:

$$\begin{array}{ll} \min & \nabla f(x)^T d + \frac{1}{2} d^T H d \\ \text{s.t.} & g_j(x) + \nabla g_j(x)^T d = 0, j \in I. \end{array} \tag{1.4}$$

If $d = 0$ and $\lambda \geq 0$, the algorithm stops. However, if $d = 0$, but $\lambda < 0$, the algorithm will not implement successfully. Recently, Z.B.Zhu [13] Consider the following QP subproblem:

$$\begin{array}{ll} \min & \nabla f(x)^T d + \frac{1}{2} d^T H d \\ \text{s.t.} & a_j(x) + \nabla g_j(x)^T d = 0, j \in L, \end{array} \tag{1.5}$$

where a_j is suitable vector, which guarantees to hold that if $d_0 = 0$, then x is a KKT point of (1.1),i.e. if $d_0 = 0$, then it holds that $\lambda_0 \geq 0$. Depended strictly on the strict complementarity, which is rather strong and difficult for testing, the superlinear convergence properties of the SQP algorithm is obtained. In addition, another some SQP algorithms (see [14]-[16]) have been proposed, the most advantage of these methods is that the superlinear convergence properties are still obtained under weaker conditions without the strict complementarity.

In this paper, we will develop an improved SQP method based on the one in [13], the direction d_0^k is only necessary to solve a equality constrained quadratic programming, which is very similar to (1.5),In order to void the Maratos effect, combined the generalized projection technique, a height-order correction direction is computed by an explicit formula, and it plays a important role in avoiding the strict complementarity. Furthermore, its global and superlinear convergence rate are obtained under some suitable conditions.

The remainder of this paper is organized as follows: The proposed algorithm is stated in section 2. In section 3, global convergence is established. Rate of superlinear convergence is analyzed in section 4.

2. Description of Algorithm

The active constraints set of (1.1) is denoted as follows:

$$I(x) = \{j \in I \mid g_j(x) = 0\}, I = \{1, 2, \ldots, m\}. \tag{2.1}$$

Throughout this paper, following basic assumptions are assumed.

H 2.1. *The feasible set $X \neq \phi$, and functions $f, g_j (j \in I)$ are twice continuously differentiable.*

H 2.2. *$\forall x \in X$, the vectors $\{\nabla g_j(x), j \in I(x)\}$ are linearly independent.*

Firstly, for a given point $x^k \in X$, by using the pivoting operation, we obtain an approximate active $J_k = J(x^k)$, such that $I(x^k) \subseteq J_k \subseteq I$.

Sub-algorithm A:

Step 1. For the current point $x^k \in X$, set $i = 0, \epsilon_i(x^k) = \epsilon_0 \in (0, 1)$.

Step 2 If $\det(A_i(x^k)^T A_i(x^k)) \geq \epsilon_i(x^k)$, let $J_k = J_i(x^k), A_k = A_i(x^k), i(x^k) = i$, STOP. Otherwise goto Step 3, where

$$J_i(x^k) = \{j \in I \mid -\epsilon_i(x^k) \leq g_j(x^k) \leq 0\}, A_i(x^k) = (\nabla g_j(x^k), j \in J_i(x^k)). \tag{2.2}$$

Step 3 Let $i = i + 1, \epsilon_i(x^k) = \frac{1}{2}\epsilon_{i-1}(x^k)$, and goto Step 2.

Theorem 2.1. [16] *For any iteration, there is no infinite cycle for above subalgorithm A. Moreover, if* $\{x^k\}_{k\in K} \to x^*$, *then there exists a constant* $\overline{\varepsilon} > 0$, *such that* $\varepsilon_{k,i_k} \geq \overline{\varepsilon}$, *for* $k \in K, k$ *large enough.*

Now, the algorithm for the solution of the problem (1.1) can be stated as follows.

Algorithm A:

Step 0 Initialization:

Given a starting point $x^0 \in X$, and an initial symmetric positive definite matrix $H_0 \in R^{n\times n}$. Choose parameters $\varepsilon_0 \in (0, 1), \alpha \in (0, \frac{1}{2}), \tau \in (2, 3)$. Set $k = 0$;

Step 1 For x^k, compute $J_k = J(x^k), A_k = A(x^k)$ by using Sub-algorithm A.

Step 2 Computation of the vector a^k :

2.1 Reorder the rows of A_k by finding its a maximal linearly independent rows subset, and denote

$$A_k \triangleq \begin{pmatrix} A_k^1 \\ A_k^2 \end{pmatrix},$$

where A_k^1, which is invertible, is the matrix whose rows are $|J_k|$ linearly independent rows of A_k ,and A_k^2 is the matrix whose rows are the remaining $n - |J_k|$ rows of A_k. Correspondingly, let $\nabla f(x^k)$ be decomposed as $\nabla f_1(x^k)$ and $\nabla f_2(x^k)$, i.e.,

$$\nabla f(x^k) \triangleq \begin{pmatrix} \nabla f_1(x^k) \\ \nabla f_2(x^k) \end{pmatrix}.$$

2.2 Solve the following system of linear equations:

$$A_k^1 u = -\nabla f_1(x^k). \tag{2.3}$$

Let $a^k = (a_j^k, j \in J_k) \in R^{|J_k|}$ be the unique solution;

Step 3 Computation of the direction d_0^k:

Solve the following equality constrained QP subproblem at x^k:

$$\begin{array}{ll} \min & \nabla f(x^k)^T d + \frac{1}{2} d^T H_k d \\ \text{s.t.} & p_j^k + \nabla g_j(x^k)^T d = 0, j \in J_k. \end{array} \tag{2.4}$$

Where

$$p_j^k = \begin{cases} -a_j^k, & a_j^k < 0, \\ g_j(x^k), & a_j^k \geq 0. \end{cases} \qquad p^k = (p_j^k, j \in J_k).$$

Let d_0^k be the KKT point of (2.4), and $v^k = (v_j^k, j \in J_k)$ be the corresponding multiplier vector. If $d_0^k = 0$, STOP. Otherwise, CONTINUE;

Step 4 Computation of the feasible direction with descent d^k :

$$d^k = d_0^k - \delta_k A_k (A_k^T A_k)^{-1} e_k. \tag{2.5}$$

Where $e_k = (1, \cdots, 1)^T \in R^{|J_k|}$, and

$$\delta_k = \frac{\|d_0^k\|(d_0^k)^T H_k d_0^k}{2|(\mu^k)^T e_k| \cdot \|d_0^k\| + 1}, \qquad \mu^k = -(A_k^T A_k)^{-1} A_k^T \nabla f(x^k). \tag{2.6}$$

Step 5 Computation of the high-order revised direction $\widetilde{d}^k$:

$$\widetilde{d}^k = -\delta_k A_k (A_k^T A_k)^{-1} (\|d_0^k\|^\tau e_k + \widetilde{g}_{J_k}(x^k + d^k)). \tag{2.7}$$

Where $\tau \in (2, 3)$, and

$$\widetilde{g}_{J_k}(x^k + d^k) = g_{J_k}(x^k + d^k) - g_{J_k}(x^k) - \nabla g_{J_k}(x^k)^T d^k. \tag{2.8}$$

Step 6 The line search:

Compute t_k, the first number t in the sequence $\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\}$ satisfying

$$f(x^k + td^k + t^2\widetilde{d}^k) \leq f(x^k) + \alpha t \nabla f(x^k)^T d^k, \tag{2.9}$$

$$g_j(x^k + td^k + t^2\widetilde{d}^k) \leq 0, \ \ j \in I. \tag{2.10}$$

Step 7 Update:

Obtain H_{k+1} by updating the positive definite matrix H_k using some quasi-Newton formulas. Set $x^{k+1} = x^k + t_k d^k + t^2 \widetilde{d}^k$, and $k = k + 1$. Go back to step 1.

3. Global Convergence of Algorithm

In this section, firstly, it is shown that Algorithm A given in section 2 is well-defined, that is to say, it is possible to execute all the steps defined above.

Lemma 3.1. *For the QP subproblem (2.4) at x^k, if $d_0^k = 0$, then x^k is a KKT point of (1.1). If $d_0^k \neq 0$, then d^k computed in step 4 is a feasible direction with descent of (1.1) at x^k.*

Proof. By the KKT conditions of QP subproblem (2.4) , we have

$$\begin{aligned} &\nabla f(x^k) + H_k d_0^k + A_k v^k = 0, \\ &p_j^k + \nabla g_j(x^k)^T d_0^k = 0, \qquad\qquad j \in J_k, \end{aligned} \tag{3.1}$$

If $d_0^k = 0$, we have,

$$\begin{aligned} &\nabla f_1(x^k) + A_k^1 v^k = 0, \nabla f_2(x^k) + A_k^2 v^k = 0, \\ &p_j^k = 0, \ a_j^k \geq 0, j \in J_k. \end{aligned} \tag{3.2}$$

Thereby, from (2.3), the fact A_k^1 is nonsingular implies that

$$v^k = a^k \geq 0.$$

In a word, we get that

$$\begin{aligned} &\nabla f(x^k) + A_k v^k = 0, \\ &g_j(x^k) = 0, v_j^k \geq 0, j \in J_k, \end{aligned} \tag{3.3}$$

let $v_j^k = 0, j \in I \setminus J_k$, which shows that x^k is a KKT point of (1.1).

If $d_0^k \neq 0$,

$$\begin{array}{rcl} g_{J_k}(x^k)^T d^k & = & A_k^T d^k = A_k^T d_0^k - \delta_k e_k = -p^k - \delta_k e_k. \\ \nabla f(x^k)^T d_0^k & = & -(d_0^k)^T H_k d_0^k + b^{kT} A_k^T d_0^k = -(d_0^k)^T H_k d_0^k + b^{kT} p^k, \\ \nabla f(x^k)^T d^k & = & \nabla f(x^k)^T d_0^k - \delta_k \nabla f(x^k)^T A_k (A_k^T A_k)^{-1} e_k = -(d_0^k)^T H_k d_0^k + b^{kT} p^k + \delta_k \mu^{kT} e_k \\ & \leq & -\frac{1}{2}(d_0^k)^T H_k d_0^k + b^{kT} p^k \leq -\frac{1}{2}(d_0^k)^T H_k d_0^k \end{array}$$

Thereby, d^k is a feasible direction with descent of (1.1) at x^k.

Lemma 3.2. *The line search in step 6 yields a stepsize $t_k = \left(\frac{1}{2}\right)^i$ for some finite $i = i(k)$.*

Proof. It is a well-known result according to Lemma 3.1. For (2.9),

$$\begin{aligned} s &\triangleq f(x^k + td^k + t^2\widetilde{d}^k) - f(x^k) - \alpha t \nabla f(x^k)^T d^k \\ &= \nabla f(x^k)^T (td^k + t^2\widetilde{d}^k) + o(t) - \alpha t \nabla f(x^k)^T d^k \\ &= (1-\alpha) t \nabla f(x^k)^T d^k + o(t) \end{aligned}$$

For (2.10),if $j \notin I(x^k)$, $g_j(x^k) < 0$; $j \in I(x^k)$, $g_j(x^k) = 0$, $\nabla g_j(x^k)^T d^k < 0$, so we have

$$g_j(x^k + td^k + t^2\widetilde{d}^k) = \nabla f(x^k)^T (td^k + t^2\widetilde{d}^k) + o(t) = \alpha t \nabla g_j(x^k)^T d^k + o(t)$$

The above discussion has shown the well-definition of Algorithm A.

In the sequel, the global convergence of Algorithm A is shown. For this reason, we make the following additional assumption.

H 3.1. *$\{x^k\}$ is bounded, which is the sequence generated by the algorithm , and there exist constants $b \geq a > 0$, such that $a||y||^2 \leq y^T H_k y \leq b||y||^2$, for all k and all $y \in R^n$.*

Since there are only finitely many choices for sets $J_k \subseteq I$, and the sequence $\{d_0^k, d_1^k, \widetilde{d}^k, a^k, v^k\}$ is bounded, we can assume without loss of generality that there exists a subsequence K, such that

$$x^k \to x^*, H_k \to H_*, d_0^k \to d_0^*, d^k \to d^*, \widetilde{d}^k \to \widetilde{d}^*, a^k \to a^*, v^k \to v^*, J_k \equiv J \neq \emptyset, k \in K, \quad (3.4)$$

where J is a constant set.

Lemma 3.3. *Suppose that assumptions H 2.1-H 3.1 hold, then,*

1) There exists a constant $\zeta > 0$, such that $\|(A_k^T A_k)^{-1}\| \leq \zeta$;

2) $\lim_{k\to\infty} d_0^k = 0$;

3) $\lim_{k\to\infty} d^k = 0$, $\lim_{k\to\infty} \widetilde{d}^k = 0$.

Proof. 1) From Sub-algorithm A and Theorem 2.1, we have the following result.

$$det(A_*^T A_*) = \lim_{k\in K} det(A_k^T A_k) \geq \lim_{k\in K} \varepsilon_k \geq \overline{\varepsilon} > 0.$$

Thereby, the first conclusion 1) follows.

2)Suppose by contradiction that $d_0^* \neq 0$. Then, from Lemma 3.1, it is obvious that d^* is well-defined, and it holds that

$$\nabla f(x^*)^T d^* < 0, \nabla g_j(x^*)^T d^* < 0, j \in I(x^*) \subseteq J. \quad (3.5)$$

Thus, from (3.5), it is easy to see that the step-size t_k obtained in step 6 are bounded away from zero on K, i.e.,

$$t_k \geq t_* = \inf\{t_k, k \in K\} > 0, k \in K. \tag{3.6}$$

In addition, from (2.9) and Lemma 3.1, it is obvious that $\{f(x^k)\}$ is monotonous decreasing. So, according to assumption H 2.1, the fact that $\{x^k\}_K \to x^*$ implies that

$$f(x^k) \to f(x^*), k \to \infty. \tag{3.7}$$

So, from (2.9), (3.5), (3.6), it holds that

$$0 = \lim_{k\in K}\left(f(x^{k+1}) - f(x^k)\right) \leq \lim_{k\in K}\left(\alpha t_k \nabla f(x^k)^T d^k\right) \leq \frac{1}{2}\alpha t_* f(x^*)^T d^* < 0, \tag{3.8}$$

which is a contradiction thus $\lim\limits_{k\to\infty} d_0^k = 0$.
3) The proof of 3) is elementary from the result of 2),1) as well as formulas (2.5) and (2.7)

Theorem 3.4. *The algorithm either stops at the KKT point x^k of the problem (1.1) in finite number of steps, or generates an infinite sequence $\{x^k\}$ any accumulation point x^* of which is a KKT point of the problem (1.1).*

4. The Rate of Convergence

In this section, we will discuss the convergent rate of the algorithm, and prove that the sequence $\{x^k\}$ generated by the algorithm is one-step superlinearly convergent under some mild conditions without the strict compementarity. For this purpose, we add some regularity hypothesis.

H 4.1. *The sequence $\{x^k\}$ generated by Algorithm A is bounded, and possess an accumulation point x^*, such that the KKT pair (x^*, u^*) satisfies the strong second-order sufficiency conditions, i.e.,*

$$d^T \nabla^2_{xx} L(x^*, u^*) d > 0, \forall\ d \in \Omega \triangleq \{d \in R^n : d \neq 0, \nabla g_{I^+}(x^*)^T d = 0\},$$

where, $L(x,u) = f(x) + \sum\limits_{j\in I} u_j g_j(x)$, $I^+ = \{j \in I : u_j^* > 0\}$.

Lemma 4.1. *Let H2.1∼ H4.1 holds,* $\lim\limits_{k\to\infty} ||x^{k+1} - x^k|| = 0$. *Thereby, the entire sequence*$\{x^k\}$ *converges to* x^*, *i.e.* $x^k \to x^*, k \to \infty$.

Proof. From the Lemma 3.3, it is easy to see that

$$\lim_{k\to\infty} ||x^{k+1} - x^k|| = \lim_{k\to\infty} (\|t_k d^k + t_k^2 \widetilde{d}^k\|) \leq \lim_{k\to\infty} (\|d^k\| + \|\widetilde{d}^k\|) = 0.$$

Moreover, together with Theorem 1.1.5 in [9], it shows that $x^k \to x^*, k \to \infty$.

Lemma 4.2. *It holds, for k large enough, that*
1) $J_k \equiv I(x^*) \triangleq I_*, a^k \to u_{I_*} = \left(u_j^*, j \in I_*\right), v^k \to \left(u_j^*, j \in I_*\right)$.
2) $||d^k|| \sim ||d_0^k||, ||\widetilde{d}^k|| = O(||d^k||^2)$,
3) $I^+ \subseteq L_k = \{j \in J_k : g_j(x^k + \nabla g_j(x^k)^T d_0^k) = 0\} \subseteq I(x^*)$.

Proof. 1)Prove $J_k \equiv I_*$.

On one hand, from Lemma 2.1, we know, for k large enough, that $I_* \subseteq J_k$. On the other hand, if it doesn't hold that $J_k \subseteq I_*$, then there exist constants j_0 and $\beta > 0$, such that

$$g_{j_0}(x^*) \le -\beta < 0, j_0 \in J_k.$$

So, according to $d_0^k \to 0$ and assumption H2, it holds, for k large enough, that

$$p_{j_0}(x^k) + \nabla g_{j_0}(x^*)^T d_0^k = \begin{cases} -a_{j_0}^k + \nabla g_{j_0}(x^*)^T d_0^k \ge -\frac{1}{2}a_{j_0}^k > 0, & a_{j_0}^k < 0, \\ g_{j_0}(x^k) + \nabla g_{j_0}(x^*)^T d_0^k \le -\frac{1}{2}\beta < 0, & a_{j_0}^k \ge 0. \end{cases}, \tag{4.1}$$

which is contradictory with (2.4) and the fact $j_0 \in J_k$. So, $J_k \equiv I_*$(for k large enough).

Prove that $a^k \to u_{I_*} = \left(u_j^*, j \in I_*\right), v^k \to \left(u_j^*, j \in I_*\right)$.

Denote $A_* = \left(\nabla g_j(x^*), j \in I_*\right)$. Reorder the rows of A_*, and mark

$$A_* \triangleq \begin{pmatrix} A_*^1 \\ A_*^2 \end{pmatrix},$$

where A_*^1, which is invertible, is the matrix whose rows are $|I_*|$ linearly independent rows of A_* ,and A_*^2 is the matrix whose rows are the remaining $n - |I_*|$ rows of A_*. Correspondingly, let $\nabla f(x^*)$ be decomposed as $\nabla f_1(x^*)$ and $\nabla f_2(x^*)$, i.e.,

$$\nabla f(x^*) \triangleq \begin{pmatrix} \nabla f_1(x^*) \\ \nabla f_2(x^*) \end{pmatrix}.$$

The fact $J_k \equiv I_*$ implies that

$$A_k^1 \to A_*^1, \nabla f_1(x^k) \to \nabla f_1(x^*), k \to \infty. \tag{4.2}$$

In addition, since x^* is a KKT point of (1.1), it is evident that

$$\nabla f(x^*) + A_* u_{I_*} = 0, u_{I_*} = -(A_*^1)^{-1} \nabla f_1(x^*). \tag{4.3}$$

Thereby, from (2.3), (4.2), and (4.3), it holds that

$$a^k \to u_{I_*}, k \to \infty.$$

While, from (2.4), the fact that $d_0^k \to 0$ implies that

$$\nabla f(x^k) + H_k d_0^k + A_k v^k = 0, v^k \to -(A_*^1)^{-1} \nabla f_1(x^*) = u_{I_*}.$$

The claim holds.

2)The proof of 2) is elementary from the formulas (2.5), (2.7) and assumption H 2.1.

3) For $\lim\limits_{k\to\infty}(x^k, d_0^k) = (x^*, 0)$, we have $L_k \subseteq I(x^*)$. Furthermore, it has $\lim\limits_{k\to\infty} u_{I^+}^k = u_{I^+}^* > 0$, so the proof is finished.

In order to obtain superlinear convergence, a crucial requirement is that a unit step size is used in a neighborhood of the solution. This can be achieved if the following assumption is satisfied.

H 4.2. *Let* $\left\|(\nabla_{xx}^2 L(x^k, u_{J_k}^k) - H_k)d^k\right\| = o\left(||d^k||\right)$, *where* $L(x, u_{J_k}^k) = f(x) + \sum\limits_{j\in J_k} u_j^k g_j(x)$.

According to Theorem 4.2 in [16], it is easy to obtain the following results.

Lemma 4.3. *For k large enough, $t_k \equiv 1$.*

Furthermore, In a way similar to the proof of Theorem 5.2 in [2], we may obtain the following theorem:

Theorem 4.4. *Under all above-mentioned assumptions, the algorithm is superlinearly convergent, i.e., the sequence $\{x^k\}$ generated by the algorithm satisfies that*

$$||x^{k+1} - x^*|| = o(||x^k - x^*||).$$

5. Numerical experiments

In this section, we carry out numerical experiments based on the Algorithm A. The code of the proposed algorithm is written by using MATLAB 7.0 and utilized the optimization toolbox. The results show that the algorithm is very effective. During the numerical experiments, it is chosen at random some parameters as follows: $\varepsilon_0 = 0.5, \alpha = 0.25, \tau = 2.25, and H_0 = I$, the $n \times n$ unit matrix.H_k is updated by the BFGS formula [10]. In the implementation, the stopping criterion of Step 1 is changed to $If \ \|d_0^k\| \leq 10^{-8} \quad STOP$.

This algorithm has been tested on some problems from Ref.[17], a feasible initial point is either provided or obtained easily for each problem. The results are summarized in Table 1. The columns of this table has the following meanings:
No.: the number of the test problem in [17];
n : the number of variables;
m: the number of inequality constraints;
NT: the number of iterations;
FV: the final value of the objective function.

Table 1

NO.	n,m	NT	$\|d_0^k\|$	FV
30	3, 7	14	5.604630296280 E-09	0.999999929472
43	4, 3	21	5.473511535838 E-09	-43.999999999998
66	3, 8	12	8.327832675386 E-09	0.518163274180
100	7, 4	18	8.595133692328 E-09	680.630057374463
113	10, 8	45	6.056765632745 E-09	24.306209068269

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