AMO - Advanced Modeling and Optimization, Volume 10, Number 2, 2008

Maximum Entropy and Primal-dual geometric programming approach in Multi-objective Vendor selection Problem

Bablu Samanta

Department of Engineering Science,

Haldia Institute of Technology, Haldia, Midnapore (East) -721657, West Bengal.,India.

Abstract: The objective of this paper is to analyze entropy based Vendor Selection Problem (VSP) model by using fuzzy mathematical programming and primal-dual geometric programming method. Here we have extended the unconstrained convex programming approach to solve VSP model with inequality constraints without adding artificial variables. By t-norm based fuzzy mathematical programming technique and the duality theory, one can solve a given model by solving the geometric dual of a perturbed VSP model.

Keywords: Multi-objective model, t-norm, Entropy, Geometric Programming.

Corresponding author: Fax +91 3224252800

E-mail: bablus@rediffmail.com

***AMO* - Advanced Modeling and Optimization. ISSN: 1841-4311**

1. Introduction:

Vendor selection is the process of choosing a partner or supplier to work with on the process that will be outsourced. It is possibly the most critical part of the entire road to outsourcing because, after months of planning, you have taken the decision to go ahead with the project and that plan must work. Vendor selection problem (VSP) is an area of tremendous importance in the effective management of a supply chain. This is due to the compelling need to evolve strategic alliances with the vendors. The material and equipment supplied from the vendors play an important role in the management of a supply chain. Many issues in the supply chain are influenced by the proper selection of vendors. In the logistics decisions of a firm, the location of vendors has a great influence on the supply chain design in terms of transportation and distribution planning. Hence, it is important to select the potential vendors so that different objectives of the supply chain are achieved. Similarly, reliable vendors may lead to less number of vendors in a supply chain, whereas the selection of a large number of vendors may be done to minimize the risk associated with the purchase, the associated costs increase with this approach. Hence, the optimization of vendor-base is needed to identify better performing vendors in a supply chain.

Dickson [5] offered a profound study of vendor selection. This work was based on a survey of purchasing managers and he ranked, in order of importance, 23 criteria for vendor selection. Weber et al. [23,24,25] offers a comprehensive review of the vendor selection criteria. Buffa and Jackson [4] formulated a goal program. However, they were unable to make order quantity allocations. Talluri [20] proposed a buyer–seller game model for purchasing and the negotiation of bids. This model effectively evaluates alternative bids based on the ideal targets set by the evaluates alternative bids based on the ideal targets set by the buyer. Mathematical programming approaches have been extensively used for the VSP. They include linear programming, mixed integer programming and goal programming etc. Moore and Fearon [13] described the possible use of the linear programming (LP) but did not present the mathematical formulation. Pan [14] developed a single item LP model to allocate order quantities of suppliers with the objective to minimize aggregate price on the constraints of quality, service level and lead-time. Turner [21] described the LP for the multiple item problem of British coal. Hong and Hayya [9] structured the VSP as a non-linear programming problem. Ghodsypour and O'Brien [8] developed a mixed integer non-linear programming model to solve a multiple sourcing problem, which considers total cost of logistics with constraints on budget, quality, service, etc. Gao and Tang [7] suggested a multi-objective model for the problem. It established a multi-objective linear programming model (MOLP) for the special issues of purchasing these raw materials, and indicates selecting items, selecting vendors and deciding ordering quantity as the key issues in optimizing purchasing policies. Generally speaking, both of these models are application-specific and thus not readily transferable, or are limited in terms of the scope of their assumptions and the set of selection criteria.

The maximum-entropy principle initiated by Jaynes'[10] is a powerful optimization technique of determining the distribution of random system in the case of partial or incomplete information or data available about the system. This principle has now been broadened and extended and has found wide applications in different fields of science and technology (Wilson [26]; Templeman and Li [19]; Kapur [11],[12]). Samanta et. al.[17] developed entropy based Transportation problem using geometric programming method. Tsao et. al. [22] introduced a linear programming with inequality constraints via entropic perturbation.

In conventional mathematical programming, the coefficient or parameters of mathematical models are assumed to be deterministic and fixed. But, there are many situations where they may not be exactly known i.e., they may be somewhat uncertain in nature. Thus the decision-making methods under uncertainty are needed. The fuzzy programmings have been proposed from this viewpoint. In decision-making process, first Bellman and Zadeh [1] introduced fuzzy set theory. Tanaka et al. [18] applied the concepts of fuzzy sets to decisions making problems by considering the objectives as fuzzy goals and Zimmermann [28] showed the classical algorithms could be used to solve multi-objective fuzzy linear programming problems. The non-linear optimization problems have been solved by various non-linear optimization techniques. Among those techniques, geometric programming (GP) is an efficient and effective method to solve a particular type of non-linear problems. Duffin,Peterson and Zener [6] , Braighter and

Philips[3] developed geometric programming to solve a class of problems called Posynomial problems.

This paper deals with a dual convex programming approach to solve a multi-objective VSP model with inequality constraints through entropic perturbation. Using t-norm based fuzzy mathematical programming technique and by applying duality theory the given multi objective model is solved by solving the geometric dual of the VSP model.

2. Mathematical Model

A multi-objective Vendor Selection Problem is considered under the following assumptions and notations:

n = total number of Vendors compelling for selection,

x_j = number of order quantity given to the vendor j,

D = Aggregate demand of the item over a fixed planning period,

P = Least total purchasing value that a vendor can have,

P_j = Price of a unit item of the ordered quantity x_j to the vendor j,

r_j = Percentage of the rejected units delivered by the vendor j,

l_j = Percentage of the late delivered units by the vendor j,

u_j = Upper limit of the quantity available for vendor j,

v_j = Vendor rating value for vendor j ,

b_j= Budget constraint allocated to each vendor,

A multi-objective Vendor selection problem with minimization of the net cost for ordering the aggregate demand, minimization of the rejected items of the vendors, minimizes the late delivered items of the vendors and at the same time maximally unbiased about the information (i.e. maximum entropy objective function) under the restrictions due to the aggregate demand , maximum capacity of the vendors, total item purchasing value constraint and budget amount allocated to the vendors for supplying the items can be stated as:

Minimize $N(x) = \sum_{j=1}^{n} p_j x_j$ (Net cost for ordering the aggregate demand objective)

Minimize $R(x) = \sum_{j=1}^{n} r_j x_j$ (Rejected items objective)

Minimize L(x) = $\sum_{j=1}^{n} l_j x_j$ (Late delivered items of the vendors objective)

Maximize E(x)= $-\sum_{j=1}^{n} x_j \ln x_j$ (Entropy objective)

subject to $\sum_{j=1}^{n} x_j = \text{D}$ (Investment restrictions)

$x_j \le u_j$, j =1,2,......,n. (Maximum capacity restrictions)

$\sum_{j=1}^{n} v_j x_j \ge P$, (Total item purchasing value constraint)

$p_j x_j \le b_j$, j =1,2,......,n. (Budget Constraint)

$x_j \ge 0$, j =1,2,......,n.

3. Basic Definitions

Fuzzy sets first introduced by Zadeh [27] in 1965 as a mathematical way of representing impreciseness or vagueness in everyday life.

Fuzzy Set: A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$. Here $\mu_{\tilde{A}} : X \to [0, 1]$ is a mapping called the membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set $\tilde{A}$. The larger $\mu_{\tilde{A}}(x)$ is the stronger the grade of membership form in $\tilde{A}$.

Convex Fuzzy Set: A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for all x_1, x_2 in X,

$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda) x_2) \ge min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ when $0 \le \lambda \le 1$.

Aggregation operators:

When the rules in the decision support system contains more than one antecedent, the degrees of strength of the antecedents need to be combined to determine the overall strength of the rule consequent. In the language of fuzzy sets, the membership values of the linguistic variables in the rule antecedents have to be combined using an aggregation operator. Formally, a general aggregation operator is a real function $T : [0,1]^n \to [0,1]$, non-decreasing in all arguments, with the properties T(0) = 0 and T(1)=1.

General aggregation operators display the whole range of behavior, disjunctive, conjunctive, averaging, mixed, commutative, mutually reinforcing or otherwise, and correspond to vague and loosely defined "and" and "or" connectives etc. Triangular norms and conorms and averaging operators are well known examples of the aggregation operators. Different classes of aggregation operators display substantially different behavior, it is not logical to use any particular class to provide generic representation of aggregation. Therefore, we will use general aggregation operators to model aggregation of rule antecedents in decision support systems. They will provide the highest degree of adaptability and excellent empirical fit. However, if there are strong reasons to restrict the selection to a particular family of operators, we will impose the relevant constraints.

Consider general aggregation operator. The function can have a simple algebraic form, such as

$$\mathrm{T}(x_1, x_2, \ldots\ldots., x_n) = \min\{x_1, x_2, \ldots\ldots\ldots., x_n\}$$

or

$$T(x_1, x_2, \ldots\ldots\ldots., x_n) = x_1 x_2 \ldots\ldots\ldots x_n = \prod_{i=1}^{n} x_i$$

or $$T(x_1, x_2, \ldots\ldots\ldots., x_n) = \min\{1, \sum_{i=1}^{n} x_i\}$$

or $$T(x_1, x_2, \ldots\ldots\ldots., x_n) = \frac{\sum_{i=1}^{n} x_i}{n}$$

The degrees of importance of rule antecedents (vector a) can be easily incorporated into aggregation operators in a variety of ways. For example

$$T(x_1, a_1; x_2, a_2, \ldots\ldots\ldots., x_n, a_n) = \min\{x_1, a_1\} \times \min\{x_2, a_2\} \ldots\ldots\ldots \times \min\{x_n, a_n\}$$

or $T(x_1, a_1; x_2, a_2, \ldots\ldots\ldots., x_n; a_n) = \min\{x_1 a_1 + x_2 a_2 + \ldots\ldots\ldots x_n a_n, 1\}$

In this article, decision making method used by the (weighted) bounded sum operator (member of Yager family of triangular conorms).

4. Fuzzy programming technique to solve MONLP problem

A Multi-Objective Non-Linear Programming (MONLP) or a Vector Minimization Problem (VMP) may be taken in the following form:

Minimize $f(x) = [\, f_1(x), f_2(x), \ldots\ldots, f_k(x)\,]^T$ (4.1)

subject to $x \,\varepsilon X = \{ x \,\varepsilon R^n : g_j(x) \le \text{ or } = \text{ or } \ge b_j$ for $j = 1, \ldots, m$; $x \ge 0 \}$.

and $l_i \le x_i \le u_i$ (i=1,2,..,n).

Zimmermann [28] showed that fuzzy programming technique could be used nicely to solve the multi-objective programming problem.

To solve the MONLP (4.1) problem, following steps are used:

Step 1: Solve the MONLP (4.1) as a single objective non-linear programming problem using only one objective at a time and ignoring the others. These solutions are known as ideal solutions.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$
\begin{array}{c} \\ x^1 \\ x^2 \\ \cdots \\ x^k \end{array}
\begin{array}{c}
\begin{array}{cccc} f_1(x) & f_2(x) & \cdots\cdots & f_k(x) \end{array} \\
\begin{bmatrix}
f_1(x^1) & f_2(x^1) & \cdots\cdots & f_k(x^1) \\
f_1(x^2) & f_2(x^2) & \cdots\cdots & f_k(x^2) \\
\cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\
f_1(x^k) & f_2(x^k) & \cdots\cdots & f_k(x^k)
\end{bmatrix}
\end{array}
$$

Here $x^1, x^2, \ldots, x^k$ are the ideal solutions of the objectives $f_1(x), f_2(x), \ldots, f_k(x)$ respectively. So $U_r = \max\{ f_r(x^1), f_r(x^2), \ldots, f_r(x^k) \}$ (4.2)

and $L_r = \min\{ f_r(x^1), f_r(x^2), \ldots, f_r(x^k) \}$ (4.3)

[L_r and U_r are lower and upper bounds of the r^{th} objective function $f_r(x)$ for $r = 1,\ldots,k$].

Step 3: Using aspiration levels of each objective of the MONLP (4.1) may be written as follows:

Find x so as to satisfy

$$f_r(x) \underset{\sim}{\leq} L_r \quad (r = 1, 2, \ldots, k) \tag{4.4}$$

$$x \in X$$

Here objective functions of (4.1) are considered as fuzzy constraints. This type of fuzzy constraints can be quantified by eliciting a corresponding membership function

$$
\left.
\begin{aligned}
\mu_r^{\omega_r}(f_r(x)) &= 0 && \text{if } f_r(x) \geq U_r \\
&= \omega_r \mu_r^1(x) && \text{if } L_r^1 < f_r(x) < U_r \\
&= \omega_r && \text{if } f_r(x) \leq L_r^1 \\
\text{where } L_r^1 = L_r + \in_r \text{ and } 0 \leq \in_r \leq U_r - L_r. &&&
\end{aligned}
\right\} (r = 1, 2, \ldots, k) \tag{4.5}
$$

Here $\mu_r^1(x)$ is a strictly monotonic decreasing function with respect to $f_r(x)$.

Having elicited the membership functions (as in (4.5)) $\mu_r^{\omega_r}$ ($f_r(x)$) for r =1,2,..,k, a general aggregation function

$$\mu_{\tilde{D}}^{\omega}(x)=F\left(\mu_1^{\omega_1}(f_1(x)),\mu_2^{\omega_2}(f_2(x)),.....,\mu_k^{\omega_k}(f_k(x))\right) \text{ is introduced.}$$

So a fuzzy multi-objective decision making problem can be defined as

$$\underset{x\in X}{Maximize}\quad \mu_{\tilde{D}}^{\omega}(x) \qquad (4.6)$$

Fuzzy decision making method used by the (weighted) bounded sum operator (member of Yager family of triangular conorms), the problem (4.6) is reduced to

$$\text{Maximize}\quad \mu_{\tilde{D}}^{\omega}(x;w)=\sum_{r=1}^{k} w_r\mu_r^{\omega_r}(f_r(x)) \qquad (4.7)$$

subject to

$$x\in X$$

$$0\le\ \mu_r^{\omega_r}(f_r(x))\le w_r \quad \text{for } r = 1, 2, \ldots, k\ .$$

where $w_r \ge 0$ for all r=1,2,...,k , $\sum_{r=1}^{k} w_r = 1$.

Step 4: Solve (4.7) to get Pareto optimal solution.

Some basic definitions and three theorems on Pareto optimal solutions are introduced below.

Definition :(Complete Optimal Solution)

x^* is said to be a complete optimal solution to the MONLP (4.1) if and only if there exists x^* εX such that $f_r(x^*) \le f_r(x)$, for r = 1, 2, …, k and for all x ε X.

However, when the objective functions of the MONLP conflict with each other, a complete optimal solution does not always exist and hence the Pareto Optimality Concept arises and it is defined as follows.

Definition : (Pareto Optimal Solution)

x^* is said to be a Pareto optimal solution to the MONLP (4.1) if and only if there does not exist another x ε X such that $f_r(x^*) \le f_r(x)$ for all r =1, 2, …, k and $f_j(x) \ne f_j(x^*)$ for at least one j, j∈{1,2,..,k}.

5. Fuzzy programming technique in Multi-Objective VSP Model.

To solve above multi-objective rural development model, step-1of 4 is used. After that according to step-2 pay-off matrix is formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ x^3 \\ x^4 \end{array} \begin{array}{c} \begin{array}{cccc} N(x) & R(x) & L(x) & E(x) \end{array} \\ \begin{bmatrix} N(x^1) & R(x^1) & L(x^1) & E(x^1) \\ N(x^2) & R(x^2) & L(x^2) & E(x^2) \\ N(x^3) & R(x^3) & L(x^3) & E(x^3) \\ N(x^4) & R(x^4) & L(x^4) & E(x^4) \end{bmatrix} \end{array}$$

Now U_1, L_1; U_2, L_2 ; U_3, L_3 ;U_4, L_4 (where $L_1 \le N(x) \le U_1$, $L_2 \le R(x) \le U_2$, $L_3 \le L(x) \le U_3$ and $L_4 \le E(x) \le U_4$) are identified.

Here, for simplicity linear membership functions $\mu_N(N(x))$, $\mu_R(R(x))$, $\mu_L(L(x))$ and $\mu_E(E(x))$ for the objective functions *N(x), R(x), L(x)* and *E(x)* respectively are defined as follows:

$$\mu_N^{\omega_1}(N(x)) = \begin{cases} \omega_1 & for\, N(x) \le L_1' \\ \omega_1\left(\dfrac{U_1 - N(x)}{U_1 - L_1'}\right) & for\, L_1' < N(x) < U_1 \\ 0 & for\, N(x) \ge U_1 \end{cases}$$

$$\mu_R^{\omega_2}(R(x)) = \begin{cases} \omega_2 & for\, R(x) \le L_2' \\ \omega_2\left(\dfrac{U_2 - R(x)}{U_2 - L_2'}\right) & for\, L_2' < R(x) < U_2 \\ 0 & for\, R(x) \ge U_2 \end{cases}$$

$$\mu_L^{\omega_3}(L(x)) = \begin{cases} \omega_3 & for\, L(x) \le L_3' \\ \omega_3\left(\dfrac{U_4 - L(x)}{U_4 - L_4'}\right) & for\, L_3' < L(x) < U_3 \\ 0 & for\, L(x) \ge U_3 \end{cases}$$

$$\mu_E^{\omega_4}(E(x)) = \begin{cases} 0 & for\, E(x) \le L_4' \\ \omega_4\left(\dfrac{E(x) - L_4'}{U_4 - L_4'}\right) & for\, L_4' < E(x) < U_4 \\ \omega_4 & for\, E(x) \ge U_4 \end{cases}$$

where $L_i' = L_i + \varepsilon_i \ (i = 1,2,3,4)$, $\varepsilon_i \in (0 , U_i - L_i)$ is a real number.

Rough sketches of $\mu_N(N(x))$, $\mu_R(R(x))$ and $\mu_L(L(x))$ are shown in Figure 1

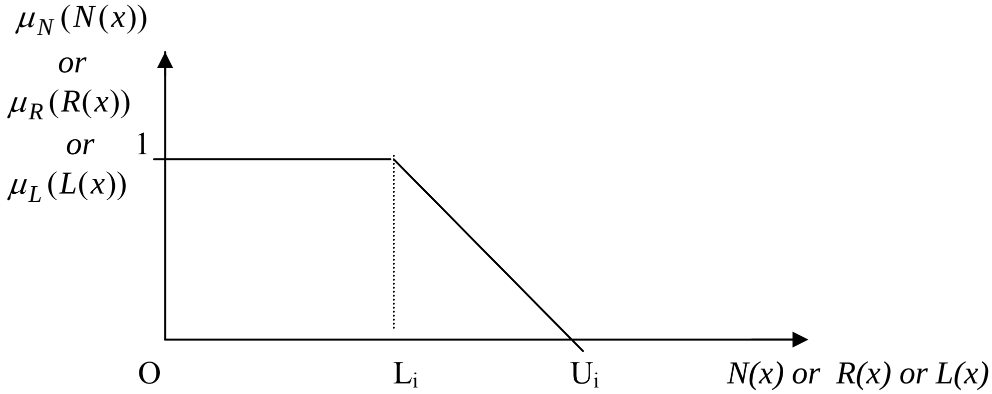

Figure - 1: Membership function for *N(x) or R(x) or L(x)* (i =1, 2,3)

Similarly rough sketch of $\mu_E(E(x))$ is shown below

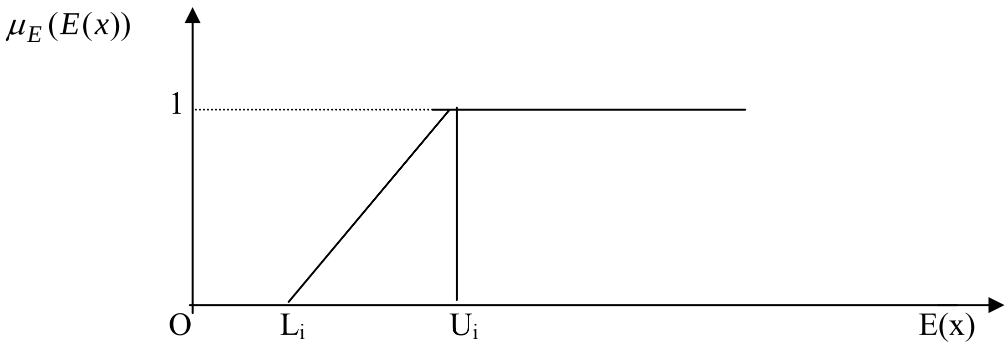

Figure - 2: Membership function for E(x) (i = 4)

According to step-3, having elicited the above membership functions crisp non-linear programming problem is formulated as follows:

Maximize $F = w_1 \mu_N^{\omega_1}(N(x)) + w_2 \mu_R^{\omega_2}(R(x)) + w_3 \mu_L^{\omega_3}(L(x)) + w_4 \mu_E^{\omega_4}(E(x))$ (5.1)

subject to

$$\mu_{\tilde{N}}^{\omega_1}(N(x)) = \omega_1\left(\frac{U_1 - N(x)}{U_1 - L_1'}\right),$$

$$\mu_R^{\omega_2}(R(x)) = \omega_2\left(\frac{U_2 - R(x)}{U_2 - L_2'}\right),$$

$$\mu_{\tilde{L}}^{\omega_3}(L(x)) = \omega_3\left(\frac{U_3 - L(x)}{U_3 - L_3'}\right),$$

$$\mu_{\tilde{E}}^{\omega_4}\left(E(x)\right) = \omega_4\left(\frac{E(x) - L_4'}{U_4 - L_4'}\right),$$

$$\sum_{j=1}^{n} x_j = \text{D}$$

$$x_j \le u_j \quad , j = 1,2,......,n.$$

$$\sum_{j=1}^{n} v_j x_j \ge P,$$

$$p_j x_j \le b_j \quad , j = 1,2,......,n.$$

$$0 \le \mu_{\tilde{N}}^{\omega_1}\left(N(x)\right) \le w_1,$$

$$0 \le \mu_{\tilde{R}}^{\omega_2}\left(R(x)\right) \le w_2,$$

$$0 \le \mu_{L}^{\omega_3}\left(L(x)\right) \le w_3,$$

$$0 \le \mu_{\tilde{E}}^{\omega_4}\left(E(x)\right) \le w_4,$$

$$x_j \ge 0, \quad j = 1,2,......,n.$$

The problem (5.1) can be written as

Maximize

$$F = \omega_1 w_1\left(\frac{U_1 - N(x)}{U_1 - L_1'}\right) + \omega_2 w_2\left(\frac{U_2 - R(x)}{U_2 - L_2'}\right) + \omega_3 w_3\left(\frac{U_3 - L(x)}{U_3 - L_3'}\right) + \omega_4 w_4\left(\frac{E(x) - L_4'}{U_4 - L_4'}\right) \tag{5.2}$$

subject to

$$\sum_{j=1}^{n} x_j = \text{D}$$

$$x_j \le u_j \quad , j = 1,2,......,n.$$

$$\sum_{j=1}^{n} v_j x_j \ge P,$$

$$p_j x_j \le b_j \quad , j = 1,2,......,n.$$

$$x_j \ge 0, \quad j = 1,2,......,n.$$

where $\omega_i \in (0,1]$ and $w_i \in [0, 1]$ for i = 1,2,3,4.

Which is equivalent to

Maximize $F' = -\alpha_1 \sum_{j=1}^{n} p_j x_j - \alpha_2 \sum_{j=1}^{n} r_j x_j - \alpha_3 \sum_{j=1}^{n} l_j x_j - \alpha_4 \sum_{j=1}^{n} x_j \ln x_j$ (5.3)

subject to the same constraints as in (5.2).

where, $\alpha_i = \dfrac{\omega_i w_i}{U_i - L_i'}$ for i = 1,2,3 ,4

and $F = F' + \dfrac{\omega_1 w_1 U_1}{U_1 - L_1'} + \dfrac{\omega_2 w_2 U_2}{U_2 - L_2'} + \dfrac{\omega_3 w_3 U_3}{U_3 - L_3'} - \dfrac{\omega_4 w_4 L_4'}{U_4 - L_4'}$.

Minimize $F' = \sum_{j=1}^{n} k_j x_j + \alpha_4 \sum_{j=1}^{n} x_j \ln x_j$, (5.4)

subject to

$$\sum_{j=1}^{n} x_j = \text{D}$$

$$x_j \le u_j \quad , j = 1,2,......,n.$$

$$\sum_{j=1}^{n} v_j x_j \ge P,$$

$$p_j x_j \le b_j \quad , j = 1,2,......,n.$$

$$x_j \ge 0 , \quad j = 1,2,......,n.$$

where $\omega_i \in (0,1]$ and $w_i \in [0, 1]$ for i = 1,2,3,4 and $k_j = \alpha_1 p_j + \alpha_2 r_j + \alpha_3 l_j$ for *j =1,2,......,n.*

6. Dual Program with Entropic Perturbation:

Consider the following linear Programming problem:

Problem 1: Minimize $\sum_{i=1}^{n} c_i x_i$

subject to $\sum_{i=1}^{m} u_i^k x_i \le v_k$, k = 1,2,…………,m.

$x_i \ge 0$, i =1,2,...,n.

The dual of the above *problem 1* is

Problem 2: Maximize $\sum_{k=1}^{m} v_k y_k$

subject to $\sum_{k=1}^{m} u_i^k y_k \leq c_i$, i =1,2,...,n.

$y_k \leq 0$, k = 1,2,…………,m,

Now for any given scalar α > 0, consider entropic perturbed problem instead of *problem 1* is as follows:

Problem 3: Minimize $\sum_{i=1}^{n} c_i x_i + \alpha \sum_{i=1}^{n} x_i \ln x_i$

subject to $\sum_{i=1}^{m} u_i^k x_i \leq v_k$, k = 1,2,…………,m.

$x_i \geq 0$, i =1,2,……..,n.

[Note: entropy function $x_{ij} \ln x_{ij}$ is strictly convex function on [0, ∞) with the convention 0 ln 0 =0]

To derive the geometric dual of *Problem 3*, consider the following inequality:

$$\ln z \leq z - 1 \text{ for } z > 0. \tag{6.1}$$

This inequality becomes equality if and only if z = 1.Now for any α > 0, y_k (k = 1,2,…………,m) are real numbers and $x_i > 0$ (i =1,2,...,n.), we define

$z_i = \dfrac{e^{[\sum_{k=1}^{m}(u_i^k y_k - c_i)/\alpha]-1}}{x_i}$ for i =1,2,...,n. Now if $x_i > 0$ implies $z_i > 0$ so by above inequality,

$$\sum_{k=1}^{m}[(u_i^k y_k - c_i)/\alpha] - 1 - \ln x_i \leq \frac{e^{[\sum_{k=1}^{m}(u_j^k y_k - c_i)/\alpha]-1}}{x_i} - 1 \tag{6.2}$$

[This inequality is valid even if $x_i = 0$ under the definition of "0 ln(0)"= 0]

Multiplying both sides of (6.2) by $x_i > 0$, we get

$$x_i \sum_{k=1}^{m}[(u_i^k y_k - c_i)/\alpha] - x_i \ln x_i \leq e^{[\sum_{k=1}^{m}(u_i^k y_k - c_i)/\alpha]-1}$$

$$\Rightarrow x_i \sum_{k=1}^{m} (u_i^k y_k - c_i) - \alpha e^{[\sum_{k=1}^{m}(u_i^k y_k - c_i)/\alpha]-1} \le \alpha x_i \ln x_i$$

$$\Rightarrow x_i \sum_{k=1}^{m} u_i^k y_k - \alpha e^{[\sum_{k=1}^{m}(u_i^k y_k - c_j)/\alpha]-1} \le c_i x_i + \alpha x_i \ln x_i \qquad (6.3)$$

Now summing over i and j of (6.3) for i =1,2,...,n

$$\sum_{i=1}^{n} x_i \sum_{k=1}^{m} u_i^k y_k - \alpha \sum_{i=1}^{n} e^{[\sum_{k=1}^{m}(u_i^k y_k - c_i)/\alpha]-1} \le \sum_{i=1}^{n} c_i x_i + \alpha \sum_{i=1}^{n} x_i \ln x_i$$

$$\Rightarrow \sum_{k=1}^{m} (\sum_{i=1}^{n} u_i^k x_i) y_k - \alpha \sum_{i=1}^{n} e^{[\sum_{k=1}^{m}(u_i^k y_k - c_i)/\alpha]-1} \le \sum_{i=1}^{n} c_i x_i + \alpha \sum_{i=1}^{n} x_i \ln x_i \qquad (6.4)$$

In problem 3 , $x_i \ge 0$,(i =1,2,...,n .) satisfies $\sum_{i=1}^{n} u_i^k x_i \le v_k$,

(k = 1,2,…………,m,) and if $y_k \le 0$, k = 1,2,…………,m then

$$\sum_{k=1}^{m} (\sum_{i=1}^{n} u_i^k x_i) y_k \ge \sum_{k=1}^{m} v_k y_k \qquad (6.5)$$

So inequality (6.4) implies

$$\sum_{k=1}^{m} v_k y_k - \alpha \sum_{i=1}^{n} e^{[\sum_{k=1}^{m}(u_i^k y_k - c_i)/\alpha]-1} \le \sum_{i=1}^{n} c_i x_i + \alpha \sum_{i=1}^{n} x_i \ln x_i \qquad (6.6)$$

Right hand side of (6.6) is exactly the objective function of Problem 3. Now define the following dual problem of Problem 3:

Problem 4: Maximize $D_\alpha = \sum_{k=1}^{m} v_k y_k - \alpha \sum_{i=1}^{n} e^{[\sum_{k=1}^{m}(u_i^k y_k - c_i)/\alpha]-1}$

Subject to

$y_k \le 0$, k = 1,2,…………,m.

Here Problem 4 is a convex programming problem with non-positivity constraints. Problem 4 can be also derived by Lagrangian method. In Lagrangian method derivation , a change of sign in a primal constraint results in a change of range of the corresponding dual variables, this casual relationship is not apparent in the geometric programming derivation. Also the above dual problem differs from the one obtained for standard form of linear programming problem only in the extra non-positivity requirements and this derivation different from as usual geometric programming with equality and inequality constraints.

To use duality theory, inequality (6.1) becomes an equality if and only if z = 1. Hence from (6.2), $z_i = \dfrac{e^{[\sum_{k=1}^{m}(u_i^k y_k - c_i)/\alpha]-1}}{x_i} = 1$

i.e $x_i = e^{[\sum_{k=1}^{m}(u_i^k y_k - c_i)/\alpha]-1}$

Assume y_k^* (k = 1,2,…………,m.) dual feasible and x_i^* primal feasible solution then solution $x_i^* = e^{[\sum_{k=1}^{m}(u_i^k y_k^* - c_i)/\alpha]-1}$.

Hence the solution of the given model are $x_i^* = e^{[\sum_{k=1}^{2n+2}(u_i^k y_k^* - k_i)/\alpha_4]-1}$ for i = 1,2,….,n.

7. Conclusion

This study presented a dual convex programming approach to solve multi-objective VSP model, which still remains uncertain. Using t-norm based fuzzy mathematical programming method; this model with entropy objectives has been reduced to a single objective primal geometric programming problem. The theory of duality is applied to solve the given model by solving the geometric dual of the perturbed rural development model. The application of this paper is widely used multi-objective entropy optimization models with linear inequality and / or equality constraints. Multi-objective Entropy model based on t-norm optimization method may be used in various fields of engineering and sciences.

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