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IMPROVING DISCRETE MODEL REPRESENTATION OF THE FAST SYSTEMS IN THE DELTA DOMAIN

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ABSTRACT

The design of the advanced control strategies is usually based on a discrete time model of the plant. The usual approach is to use the shift operator q, but this gives numerical difficulties with the small sampling periods. This paper provides a formulation for a linear, time-invariant single-input single-output continuous-time plant model in the δ domain for two fast sampled systems with non-minimum phase zeros. This type of systems is critically unstable and requires special control strategies. The δ domain approach improves the numerical properties of structure detection, leads to a parsimonious description and provides a model that is closed linked to its continuous counterpart for fast sampling systems. The δ discrete linear model is derived in order to be of interest in control design.

KEY WORDS: modeling fast systems, δ operator, critical unstable system, synchronous generator excitation system.

1. Introduction

The δ domain models provide several advantages compared to q domain representations in system modeling, especially at fast sampled systems. Middleton and Goodwin [1] have renewed interest in the use of a δ operator to parametrize the models of systems that are fast-sampled. This leads to an improvement on the numerical ill-conditioning problems found when using shift operator q [2]. The interest for the δ operator in the academic community has been renewed in the last years and therefore has been widely investigated in problems in the areas of signal processing [3], systems modeling [4-7] and control [8-10].

An advantage of modeling linear systems in δ domain consists in the fact that it provides an exact discrete-time representation of the system [7], [11]; the identified model has structural similarity to the continuous-time differential equation describing system dynamics. Moreover, the parameters of the identified model approach the continuous time values as the sampling time tends to zero. It has been concluded that the utilization of the δ operator has the advantage to overcome the problem of numerical similarity especially in conditions of fast-sampling. It has been demonstrated that the model description in δ domain directly results in the improvement of the numerical properties of structure detection. Where digital control is to be applied, it is desirable to develop a discrete-time model for analysis purposes.

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The stability of a digital system may be lost due to the finite word length effects at practical implementation of digital control or filter algorithms [11]. In 2007, Wills and the collaborators [12] have design a software package for the estimation of dynamic systems in the delta domain. The toolbox implements several new approaches as Expectation Maximisation algorithm for computation of Maximum Likelihood estimates, the use of an adaptive Jacobian rank algorithm and the use of a delta operator model. The toolbox is able to perform identification from either time domain or frequency domain data.

In the context of the power plant systems there have been investigated the numerical properties and round off noise effects caused by finite word length in [11], [13]. An alternative for classical q discrete operator, the δ discrete model has been obtained for the synchronous generator excitation system in [14]. The other advantage of delta operator utilization for the model parametrization consists in fact that it offers substantial numerical advantages in implementation of discrete-time models due to the convenience in choosing a small value for the sampling period.

The goal of this study is to analyze and develop a δ discrete model justified by the fact that the δ operator offers high performance even with low precision representation of the model coefficients. To demonstrate that implementing a discrete-time system by the delta model has distinct advantages over the commonly adopted approach of the shift model, we propose two examples. The first one is an empirical system and the second example refers to a turbine generator connected to an infinite bus-bar. This plant has a very fast dynamic, involving very short sampling and therefore we propose an accurate model especially when short sampling periods are to be used.

The paper is organized as follows: the second part of the paper introduces the concepts of the delta operator. The next section deals with two case studies: an empirical system and a simplified dynamic model for the Synchronous Machine Infinite Bus subsystem. In order to elucidate some aspects of δ operator, the properties are investigated through the two examples with regard to convergence of the discrete-time model representation to the continuous–time system. Section 3 gives the delta discrete time linear model and examines the pole-zero location for different sampling periods. Finally the main results of the paper are summarized in Section 4.

2. Delta operator

The discrete δ operator facilitates the expression of discrete time approximations of n^{th} -order derivative information contained within sampled data, where δ is defined as:

$$\delta = \frac{q-1}{T} \tag{1}$$

Although there is a linear transformation between the two discrete domains, the two operators have distinct conceptual roles [5]:

$$\begin{aligned} q^j &= (1+\delta T_s)^j = \sum_{n=0}^{j} C_j^n (T_s\delta)^j \\ C_j^n &= \frac{j!}{n!(j-n)!} \end{aligned} \tag{2}$$

where T is the sampling period and q is the usual forward-shift operator.
The δ discrete state space model can be written:

$$\begin{aligned}\delta x(k) &= \mathbf{A}_\delta x(k) + \mathbf{B}_\delta u(k) \\ y(k) &= \mathbf{C}_\delta x(k) + \mathbf{D}_\delta u(k)\end{aligned} \tag{3}$$

The deterministic case of single input single output state space form in the classical representation in q discrete domain is:

$$\begin{aligned}qx(k) &= \mathbf{A}_q x(k) + \mathbf{B}_q u(k) \\ y(k) &= \mathbf{C}_q x(k) + \mathbf{D}_q u(k)\end{aligned} \tag{4}$$

where the link between the two discrete models [8] is given by:

$$\begin{aligned}\mathbf{A}_\delta &= \frac{\mathbf{A}_q - I}{T}, \\ \mathbf{B}_\delta &= \frac{\mathbf{B}_q}{T}, \\ \mathbf{C}_\delta &= \mathbf{C}_q, \\ \mathbf{D}_\delta &= \mathbf{D}_q\end{aligned} \tag{5}$$

Although the matrices obtained in (5) are mathematically correct, the usage of it is not recommended due to the poor numerical properties in q domain representation. Therefore, Middleton and Goodwin have proposed a procedure for obtaining delta state space model directly from the continuous model [1]. Thus, they suggest the following relations for conversion from s-domain model into the δ-domain one:

$$\begin{aligned}\mathbf{A}_\delta &= \frac{e^{\mathbf{A}_c T} - I}{T} = \Omega \mathbf{A}_c \\ \mathbf{B}_\delta &= \Omega \mathbf{B}_c \\ \mathbf{C}_\delta &= \mathbf{C}_c \\ \mathbf{D}_\delta &= \mathbf{D}_c\end{aligned} \tag{6}$$

where $\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_c, \mathbf{D}_c$ are continuous-time state space model matrices and:

$$\begin{aligned}\mathbf{\Omega} &= \frac{1}{T}\int_0^T e^{\mathbf{A}_c \tau} d\tau = \frac{1}{T}(e^{\mathbf{A}_c T} - I)\mathbf{A}_c^{-1} \\ &= I + \frac{\mathbf{A}_c T}{2!} + \frac{\mathbf{A}_c^2 T^2}{3!} + \ldots\ldots\end{aligned} \tag{7}$$

The correspondence between the two domains is emphasized in the limit case:

$$\begin{aligned}\lim_{T\to 0} \mathbf{A}_\delta &= \mathbf{A}_c \\ \lim_{T\to 0} \mathbf{B}_\delta &= \mathbf{B}_c\end{aligned} \tag{8}$$

It is easy to notice that for the limit case $T \to 0$, there is no equivalence between the matrices obtained in the q discrete time domain and s –domain as one would expect:

$$\begin{aligned}\lim_{T\to 0} \mathbf{A}_q &= \mathbf{I} \\ \lim_{T\to 0} \mathbf{B}_\delta &= 0\end{aligned} \tag{9}$$

The input-output form in δ-domain can be calculated as:

$$G(\delta) = \mathbf{C}_\delta (\delta \mathbf{I}_n - \mathbf{A}_\delta)^{-1} \mathbf{B}_\delta + \mathbf{D}_{\delta}., \tag{10}$$

and

$$G(\delta) = G(q)\,|_{q=1+T\delta} \tag{11}$$

An advantage of modeling linear systems in δ domain consists in the fact that it provides an exact discrete-time representation of the system. Moreover, the parameters of the identified model approach the continuous time values as the sampling time tends to zero. It is to be mentioned that small period results usually from the demands concerning the quality of control.

3. Case studies

This section demonstrates the performance of the δ domain parametrization via simulations for two systems with respect to sampling period. The first system used in this study is an empirical model and was previously utilized in [14] and [9] in order to demonstrate the advantages of the delta operator. The second system represents a simplified dynamic model for the Single Machine Infinite Bus subsystem (SMIB). Note that the above systems are non-minimum phase and the first case study incorporates an integrator.

3.1 Illustrative example

The proposed continuous-time domain state-space matrices are given by:

$$\begin{gathered} \mathbf{A}_c = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ \mathbf{B}_c = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \\ \mathbf{C}_c = [0 \quad -0.2 \quad 1] \end{gathered} \tag{12}$$

The associated transfer function is:

$$G(s) = \frac{-5.55 \cdot 10^{-16} s^2 - 0.2s + 1}{s^3 + s}. \tag{13}$$

The poles and zeros of the system are:

$$\begin{gathered} p_1 = 0, p_{2,3} = 0 \pm i, \\ z_1 = -3.60 \cdot 10^{14},\ \mathrm{z}_2 = 4.99 \end{gathered} \tag{14}$$

The poles and zeros placement of the system is illustrated in Fig. 1.

When using very fast sampling it has been proved that the non-minimum phase zeros will appear as sampling period decrease, even though all zeros of the continuous time system are located in the strictly left complex plane.

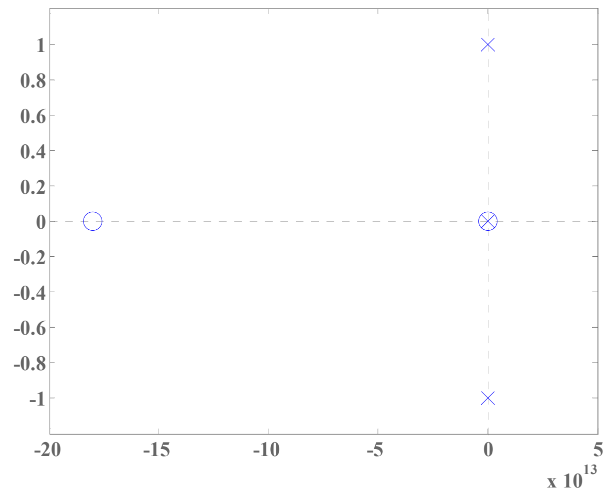

Fig. 1 The poles (x)-zeros (○) location of the continuous time transfer function

For the shift operator model, considering the sample rate $f = 1000Hz$, the transfer function is obtained:

$$G(z) = \frac{-9.98 \cdot 10^{-8} z^2 + 6.66 \cdot 10^{-10} z - 1 \cdot 10^{-7}}{z^3 - 3z^2 + 3z - 1}. \tag{15}$$

The poles and zeros of the discretized system are:

$$\begin{aligned} & p_1 = 9.99 \cdot 10^{-1},\ p_{2,3} = 9.99 \cdot 10^{-1} \pm 9.99 \cdot 10^{-4} i \\ & z_1 = 1.00,\ z_2 = -9.98 \cdot 10^{-1} \end{aligned}. \tag{16}$$

The Fig. 2 visualizes the influence of the sampling period on the location of the poles and zeros for q-model. The sample time of the discretized model was set to $0.1s,\ 0.01s$ and $0.001s$.

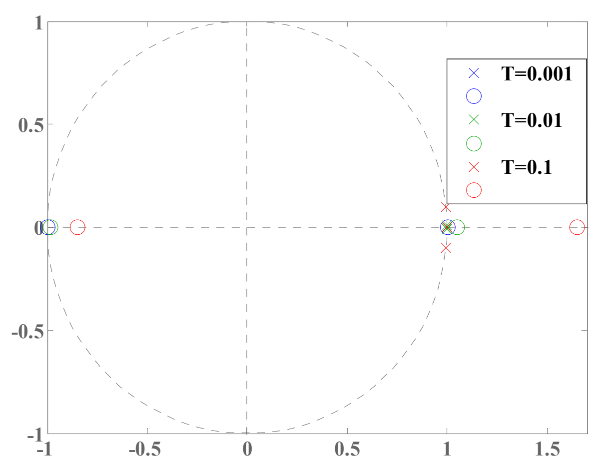

Fig. 2 The poles (x)-zeros (○) location of the q-discrete time transfer function

The sampling period influences the numerical sensitivity such that a short sampling period requires a high precision in the coefficients. This aspect is related to the increased clustering of the poles and zeros around the point (1, 0) in the q - plane [11]. This drawback can be overcame by replacing the shift model by the one based on the δ operator:

$$G(\delta) = \frac{-9.98 \cdot 10^{-5} \delta^2 - 0.19\delta + 1}{\delta^3 + 0.001\delta^2 + \delta}. \tag{17}$$

The poles and zeros of the δ domain transfer function are:

$$\begin{aligned} &p_1 = 0, p_{2,3} = -4.99 \cdot 10^{-4} \pm 9.99 \cdot 10^{-1} i \\ &z_1 = -1.99 \cdot 10^3,\ z_2 = 5.01 \end{aligned} \tag{18}$$

In the case of δ operator as sampling period decreases, all poles from δ domain tend to the continuous time poles, as is seen in the Fig. 3.

The lack of connection between the q-domain and continuous time poles is reinforced by the clustering of the dynamic information to the point (1, 0) in the z plane, when the sampling interval becomes very small.

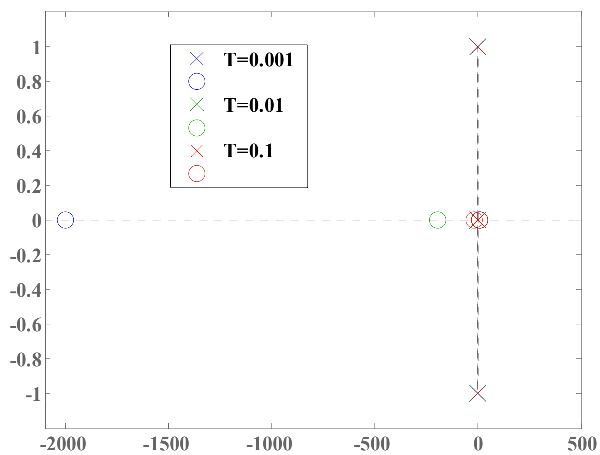

Fig. 3 The poles (x)-zeros (○)location of the δ domain model transfer function

3.2 The Mathematical Model of a Single Machine Infinite Bus

In this paragraph is presented a simplified dynamic model for the Single Machine Infinite Bus subsystem (SMIB). This system consists in a single synchronous generator connected through a parallel transmission line to a very large network approximated by an infinite bus. Synchronous generator excitation control is one of the most important measures to enhance power system stability and to guarantee the quality of electrical power.

3.2.1 Process description

The main components in a steam turbine driven alternator, feeding a main bus-bar, are shown in the Fig. 4.

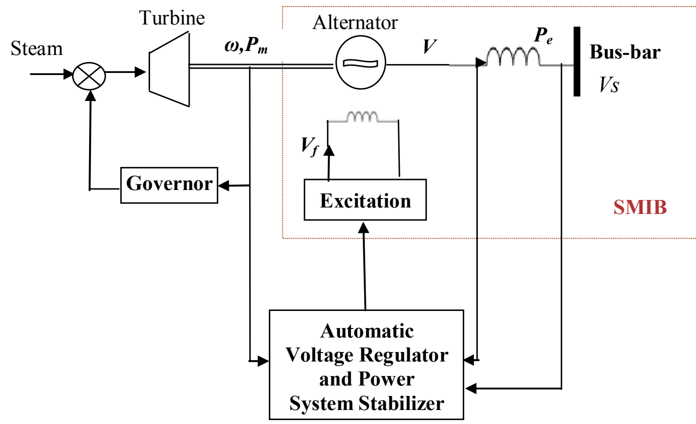

Fig. 4 Power System with steam turbine and alternator

As is shown in the Fig. 4, the most important regulation loops are the voltage control and the primary turbine speed control. The voltage control of the generator system is realized by the Automatic Voltage Regulator. The speed control system adjusts the steam flow into the turbines in response to changes in shaft speed [17]. The necessary mechanical power, P_m is provided to meet the demanded electrical load. In the Fig. 4, V means the alternator output voltage, V_f the excitation voltage, P_e electrical power and Vs the bus-bar voltage. In reality, a power station involves a number of four alternator/turbine connected to a bus-bar and the wider electrical transmission and distribution system. Each alternator has its own individual Automatic Voltage Regulator. In Fig 4, the generator must remain synchronized to the grid that provides the interconnection to other power stations and distribution centers. The synchronized torque results from the magnetic fields, which provide damping torque [18]. The mechanical power input (from the steam turbine) and the field excitation voltage can be considered to be the system inputs and the output voltage and frequency must be controlled. This multivariable system involves the control of very fast dynamic and light damped modes. It has been assumed that this type of energetic system have a very small time constant. The application of digital techniques for controlling this type of fast system has expanded in recent years with the introduction of low cost digital controller hardware. The advanced control strategies which have been proposed require discrete time models. One common way of describing discrete-time models is to use the forward shift operator. Discrete time system study is usually done using q forward shift operator and associated discrete frequency variable z. Despite its wide use in digital control, it is apparent that the rational transfer function which is obtained using the shift operator is not all like the transfer function obtained in the s-domain. To obtain a better correspondence to its continuous time counterpart model, the delta discrete time operator must be use.

3.2.2 The mathematical model

The dynamic model of the power system is based on the mechanical equations and the electrical generator dynamics [18]. In this paper only the liniarized model for a single machine to infinite bus plant is illustrated since a delta discrete model representation is of interest. For the sake of brevity and simplicity, the monovariable state space model of the SMIB is considered:

$$\begin{cases} \frac{d}{dt}x = \mathbf{A}_c x + \mathbf{B}_c u \\ y = \mathbf{C}_c x \end{cases}. \tag{19}$$

where the state vector x is defined as:

$$x = [\Delta\alpha \quad \Delta\omega \quad \Delta E]^{\sim} \tag{20}$$

$\Delta\alpha$ represents the power angle of the generator for a nominal point, $\Delta\omega$ - the rotor speed of the generator and ΔE - transient EMF in the quadratic axis of the generator.

The excitation control input $u = \Delta V_f$ is referred as input signal and the output $y = \Delta V$. More information about the state space model (19) can be found in [18]. The matrices $\mathbf{A}_c$, $\mathbf{B}_c$ and $\mathbf{C}_c$ are as follows:

$$\begin{aligned} \mathbf{A}_c &= \begin{bmatrix} 0 & 1 & 0 \\ 6.00\cdot 10^5 & -6.25\cdot 10^{-1} & 5.04\cdot 10^6 \\ 2.98\cdot 10^1 & 0 & -2.80\cdot 10^{-1} \end{bmatrix} \\ \mathbf{B}_c &= \begin{bmatrix} 0 \\ 0 \\ 1.44\cdot 10^{-1} \end{bmatrix} \\ \mathbf{C}_c &= \begin{bmatrix} -1.13\cdot 10^3 & 0 & 2.79 \end{bmatrix} \end{aligned}. \tag{21}$$

The associated transfer function is:

$$G(s) = \frac{0.40s^2 + 0.25s - 8.27\cdot 10^8}{s^3 + 0.90s^2 - 6\cdot 10^5 s - 1.50\cdot 10^8}. \tag{22}$$

The relative degree of the system is one and transfer function pole-zero location is shown in the Fig. 5.

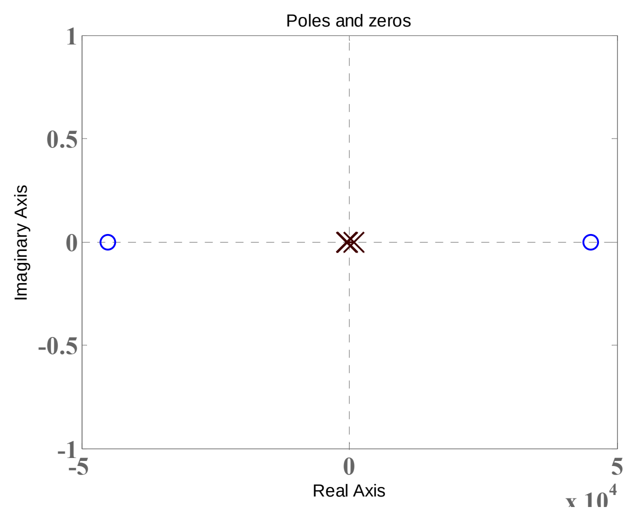

Fig. 5 The poles (x)-zeros (○) location of the continuous time transfer function

The poles and zeros of the system are:

$$p_1 = 8.78 \cdot 10^2,\ p_2 = -5.86 \cdot 10^2,\ p_3 = -2.93 \cdot 10^2$$
$$z_1 = -4.51 \cdot 10^4,\ z_2 = 4.51 \cdot 10^4 \tag{23}$$

The single machine infinite bus plant is an unstable system with one zero in the right half plane of *s* domain.

In the following we intend to compare the models obtained in the two discrete time domains. The selection of the sampling period is normally based on Shannon's reconstruction theorem. It has been proved that sampling at a rate less than ten times the bandwidth involves a loss of information regarding inter-sample behavior. Accordingly, sampling rates up to 50 times the closed loop bandwidth are sometimes chosen in fast, high precision digital control system as is illustrated in the power system application [11].

The system bandwidth is found from the 3 db points to be $\omega_0 = 590 rad/\sec$, suggesting a sampling rate in the range of $939.01 \le f \le 4695.07$. Therefore a suggestion for the choice of sampling period is given by the range: $2.12 \cdot 10^{-4} \le T \le 1.06 \cdot 10^{-3}$.

For the shift operator model, considering the interval between samples, $T = 2.12 \cdot 10^{-4}$, the transfer function is obtained:

$$G(z) = \frac{-1.2 \cdot 10^{-3} z^2 - 5.4 \cdot 10^{-3} z - 1.2 \cdot 10^{-3}}{z^3 - 3.02 z^2 + 3.02 z - 0.99} \quad . \tag{24}$$

The poles and zeros of the discrete transfer function are:

$$p_1 = 1.20,\ p_2 = 0.93,\ p_3 = 0.88$$
$$z_1 = -4.18,\ z_2 = -0.23 \tag{25}$$

Unfortunately, the discrete domains are unconnected with the continuous domain, this is because the underlying continuous domain description cannot be obtained by setting the sample time *T*=0.

Fig. 6 shows the influence of the sampling period on the location of the poles and zeros for *q*-model.

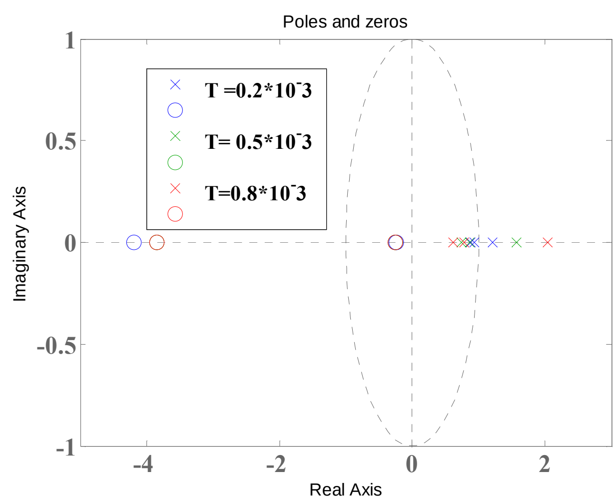

Fig. 6 The poles (x)-zeros (○)variation of shift operator model with respect to sampling period

Another convenient feature of the δ operator is that the poles and zeros approach those of the continuous time representation as T tends to zero.

$$G(\delta)=\frac{-5.79\delta^{2}-1.78\cdot 10^{5}\delta-8.29\cdot 10^{8}}{\delta^{3}-130.1\delta^{2}-6.49\cdot 10^{5}\delta-1.51\cdot 10^{8}}. \quad (26)$$

In the case of δ operator model, the poles and zeros of the discrete transfer function are:

$$\begin{aligned} &p_1=9.65\cdot 10^{2},\ p_2=-5.51\cdot 10^{2},\ \mathrm{p}_3=-2.84\cdot 10^{2} \\ &z_1=-2.44\cdot 10^{4},\ \mathrm{z}_2=-0.58\cdot 10^{4} \end{aligned} \quad (27)$$

By δ domain modeling, the zeros introduced by discretization process migrate to negative infinity as the sampling time tends to zero.

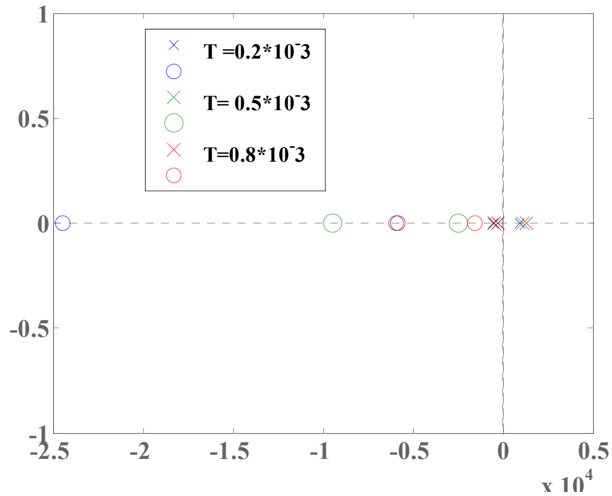

Fig. 7 The poles (X)-zeros (○) variation of the δ domain model with respect to sampling period

There is a close connection between continuous time result and δ - representation. In fact δ domain description converges to the continuous time counterpart when the sampling period tends to zero. It is of interest to compare the poles and zeros based on the δ and q discrete time single input single output system. If we compare the poles and zeros obtained in the δ discrete domain (27) with the poles and zeros in s domain (23) is observed that the values are close related. The simulation results shows the consistency of the delta model poles and zeros which correspond one-for-one with the continuous poles and zeros and converge to them as the sampling time decrease to zero.

The stability region in the δ domain depends on the sampling period, i.e. the stability region is a circle of radius $\frac{1}{T}$ and center $-\frac{1}{T}$. Thus for T=0 the stability domain covers all negative left half plane from s domain.

4. Conclusion

This paper illustrates an alternative discrete time model representation for fast sampled systems, using the delta operator. The usual approach is use the shift operator q, but this involves numerical difficulties with small sampling periods, especially for fast sampled systems. Unfortunately, high sampling rate can have adverse effect on algorithms when used with the *q* operator. In the shift form as the sampling rate increases, the poles and zeros cluster around the point (1, 0), but by δ domain modeling, the zeros migrate to negative infinity. This paper deals with the effects of the sampling time variation and illustrates two single input single outputs systems expressed in both *q*- and δ-model. A discrete time representation of a synchronous generator with excitation system has been derived in order to be of interest in control design. The simulation results demonstrate that δ domain approach improves the numerical properties of structure detection, leads to a parsimonious description and provides a model that is closed linked to its continuous counterpart for fast sampling systems. A turbine generator connected to an infinite bus-bar has a very fast dynamic and an accurate model must be elaborate especially when short sampling periods are involved. The paper emphasizes that implementing a discrete-time system by the delta model has distinct advantages over the commonly adopted approach of the shift model for short sampling intervals.

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