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Analysis of Partial Trade Credit Financing in a Supply chain by EPQ-based models

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Abstract

In the classical inventory (or) supply chain models, it was assumed that the retailers and their customers must pay for the items as soon as the items are received. However, in practice, the supplier usually is willing to provide the retailer a full trade credit period for payments and the retailer just offers the partial trade credit period to his/her customers. This paper develops economic production quantity (EPQ)- based model to investigate the retailer's inventory system in a supply chain as a cost minimization problem under partial trade credit option to their customers. Mathematical theorems are developed to determine optimal inventory policy for the retailer and numerical examples are given to illustrate the theorems. We deduce some previously published results of other researchers as particular cases and obtain lot of managerial phenomena.

Keywords: Partial trade credit; EPQ models ; Supply chain ; Optimization

1. Introduction

In today's business transactions, it is more and more common to see that retailers are allowed a fixed time period before they settle their account to the supplier. We term this period as trade credit period. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled at the end of the trade credit period. In the real world, the supplier would allow a specified credit period (say, 30 days) to the retailer for payment without penalty to stimulate the demand of consumable products. This credit term in financial management is denoted as 'net 30'. The trade credit financing produces

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two benefits to the supplier: (1) it should attract new customers who consider it to be a type of price reduction; and (2) it should cause a reduction in sales outstanding, since some established customers will pay more promptly in order to take advantage of trade credit more frequently. In India, gas stations adopted a pricing policy that charged less money per gallon to the customer who paid by cash, instead of by a credit card. Likewise, a store owner in many China towns around the world usually charges a customer 5 % more if the customer pays by a credit card, instead of by cash. As a result, the customer must decide which alternative to take when the supplier provides not only a cash discount but also a permissible delay.

One level trade credit financing refers that the supplier would offer the retailer trade credit but the retailer would not offer the trade credit to his/her customers. That is, the retailer could sell the goods and accumulate revenue and earn interest within the trade credit period but the customer would pay for the items as soon as the items are received from the retailer. Several interesting and relevant papers related to one level trade credit financing exist in the literature. Goyal (1985) analyzed effects of trade credit on the optimal inventory policy. Chand and Ward (1987) analyzed Goyal's problem under assumptions of the classical economic order quantity model, obtaining different results. Chung (1998) developed an alternative approach to determine the economic order quantity under the condition of permissible delay in payments. Aggarwal and Jaggi (1995) considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Chu et al. (1998) extended Goyal's model to the case of deterioration. Jamal et al. (1997) and Chang and Dye (2001) further generalized the model with shortages. Many related articles can be found in Hwang and Shinn (1997), Jamal et al. (2000), Arcelus et al. (2003), Abad and Jaggi (2003) and Chang (2004), Chung et al. (2005), Chung and Liao (2006), and Huang (2007) and their references.

Partial trade credit financing refers to paying partial amount for the purchased items as soon as the items are received and remaining amount should be settled at the end of a trade credit period. Before the end of the trade credit period, we can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the trade credit period. Partial trade credit financing is one of central features in supply chain management. Since in most business transactions, the one level trade credit financing is unrealistic, we want to investigate the situation in a supply chain in which the supplier is willing to provide the retailer a full trade credit period for payments and the retailer offers the partial trade credit to his/her customers. In practice, this partial trade credit financing at a retailer is more matched to real life supply chains. For example, in India, the TATA Company can delay the full amount of purchasing cost until the end of the delay period offered by his supplier. But the TATA Company only offers partial delay payment to his dealership on the permissible credit period and the rest of the total amount is payable at the time the dealership places a replenishment order.

In the above models, it was assumed that the products are obtained from an outside supplier and the entire lot size was delivered at the same time. In fact, when a product can be produced in-house, the replenishment rate is also the production rate, and is hence finite. Hence, the Economic Production Quantity (EPQ) model should be the efficient model to deal with inventory management issues in a supply chain. It is considered to be one of the most popular inventory control models used in an industry. In this paper, we complement the shortcomings of all the previously mentioned models by relaxing the traditional EPQ model in the following 4 ways: (1) the supplier is willing to provide the retailer a full trade credit period for payments and the retailer offers the partial trade credit period to his/her customers; (2) the retailer's trade credit period offered by the supplier is not necessarily longer than the customer's trade credit period offered by the retailer; (3) the retailer's selling price per unit is not necessarily higher than the purchase unit cost; (4) interest charge rate is not necessarily higher than the interest earned rate. The retailer could sell the goods and accumulate revenue and earn interest within the trade credit period offered by the supplier and retailer must pay interest to the items to the supplier if the payment is not settled at the end of trade credit period. Under these conditions, we model the retailer's inventory system as a cost minimization problem to determine retailer's optimal ordering policies.

2. Notations and assumptions

The following notations and assumptions will be used at the retailer of a supply chain.

Notations

D	demand rate per year
P	production rate per year
ρ	$1 - \frac{D}{P} \geq 0$
h	stock-holding cost per unit per year excluding interest charges
A	ordering cost per order
I_e	interest earned per \$ per year

I_k	interest charges payable per \$ per year to the supplier
c	unit purchasing price
s	unit selling price
α	customer's fraction of the total amount owed payable at the time of placing an order offered by the retailer, $0 \leq \alpha \leq 1$
M	retailer's trade credit period offered by the supplier in years
N	customer's trade credit period offered by the retailer in years
T	cycle time in years
$TC(T)$	annual total cost, which is a function of T
T^*	optimal cycle time
$Q^* = DT^*$	optimal order quantity.

Assumptions

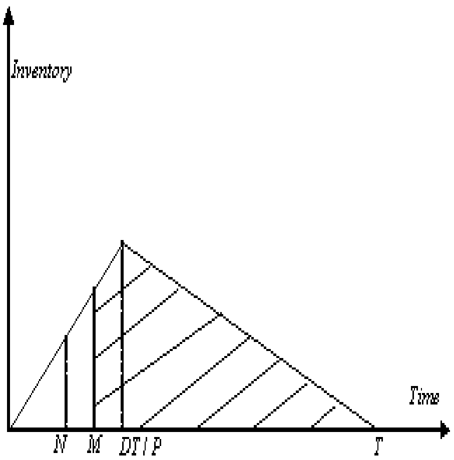
1. Replenishment rate, P, is known and uniform.
2. Demand rate, D, is known and constant.
3. The supplier offers the full trade credit to the retailer. When $T \geq M$, the account is settled at $T = M$, the retailer pays off all units sold and keeps his/her profits, and starts paying for the interest charges on the items in stock with rate I_k. When $T \leq M$, the account is settled at $T = M$ and the retailer no need to pay any interest on the stock.
4. The retailer just offers the partial trade credit to his/her customers. Hence, the customer must make a partial payment to the retailer when the item is sold. Then the customer must pay off the remaining balance at the end of the trade credit period offered by the retailer. That is, the retailer can accumulate interest from his/her customer payment with rate I_e.
5. Time horizon is infinite.
6. Shortages are not allowed.

3. Model formulation

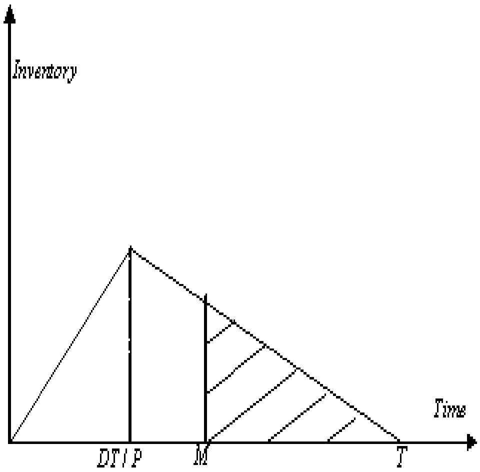
The annual total cost incurred at the retailer,

$$TC(T) = \text{ Setup cost + Holding cost + Interest payable - Interest earned}$$

Trade credit period of the retailer may be greater or lesser to the trade credit period of his customer. So, two situations may arise: (I) $M \geq N$ and (II) $M < N$.



(a)



(b)

Figure 1: (a) Total accumulation of interest payable when $PM/D \leq T$, (b) Total accumulation of interest payable when $M \leq T \leq PM/D$ and $DT/P \leq N \leq M$ (or) $N \leq DT/P$

Case I: When $M \geq N$

1. Annual ordering cost is A/T.
2. Excluding interest charges, the annual stock-holding cost is (shown in Fig.1(a))

$$\frac{hT(P-D)(DT/P)}{2T} = \frac{DTh}{2}(1-\frac{D}{P}) = \frac{DTh\rho}{2}$$

3. According to assumption (3), there are four cases that occur in costs of interest payable for the items kept in stock per year.

   (a) For $M \leq PM/D \leq T$, (shown in Fig.1 (a))

   the annual amount of interest payable

$$= \frac{cI_k}{T}\left[\frac{DT^2\rho}{2} - \frac{(P-D)M^2}{2}\right] = \frac{cI_k\rho}{T}\left[\frac{DT^2 - PM^2}{2}\right]$$

   (b) For $M \leq T \leq PM/D$, (shown in Fig. 1(b))

   the annual amount of interest payable

$$= \frac{cI_k}{T}\left[\frac{D(T-M)^2}{2}\right]$$

(c) For $N \leq T \leq M$,

the annual interest payable amount is zero.

(d) For $T \leq N$,

there is no annual interest payable cost.

4. According to assumption (4), there are four cases that occur in interest earned per year.

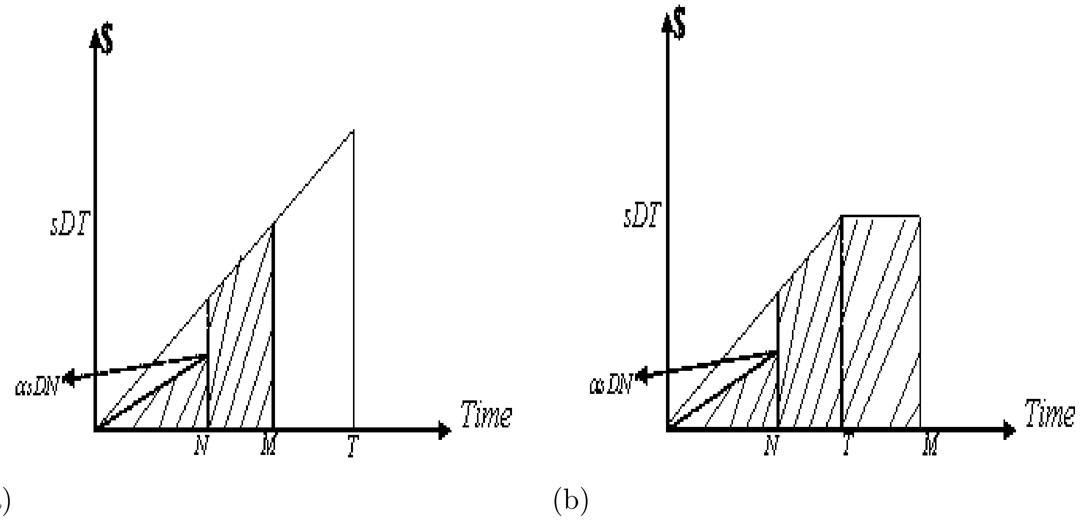


Figure 2: (a) Total amount of interest earned when $PM/D \leq T$ (or) $M \leq T \leq PM/D$, (b) Total amount of interest earned when $N \leq T \leq M$

(a) For $M \leq PM/D \leq T$ and $M \leq T \leq PM/D$ (see Fig. 2(a)), the annual interest earned is

$$\frac{sI_e}{T}\left[\frac{\alpha DN^2}{2}+\frac{(DN+DM)(M-N)}{2}\right]=\frac{sI_eD}{2T}[M^2-(1-\alpha)N^2]$$

(b) For $N \leq T \leq M$, (see Fig. 2(b)) the annual interest earned is

$$\frac{sI_e}{T}\left[\frac{\alpha DN^2}{2}+\frac{(DN+DT)(T-N)}{2}+DT(M-T)\right]=\frac{sI_eD}{2T}[2MT-(1-\alpha)N^2-T^2]$$

(c) For $T \leq N$ (see Fig. 3), the annual interest earned is

$$\frac{sI_e}{T}\left[\frac{\alpha DT^2}{2}+\alpha DT(N-T)+DT(M-N)\right]=sI_eD\left[M-(1-\alpha)N-\frac{\alpha T}{2}\right]$$

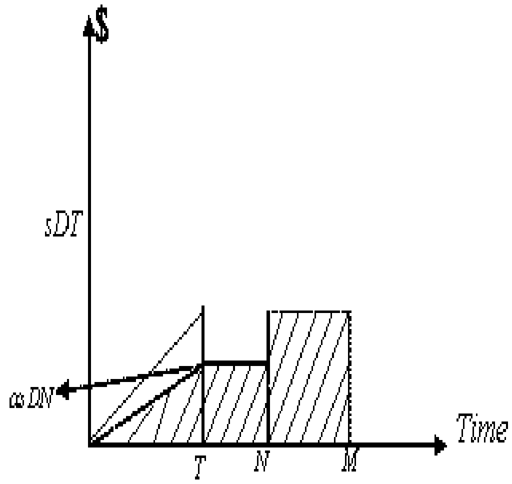


Figure 3: Total amount of interest earned when $T \leq N$

The total cost incurred at the retailer, $TC(T)$, is

$$TC(T) = \begin{cases} TC_1(T) & \text{if } PM/D \leq T \\ TC_2(T) & \text{if } M \leq T \leq PM/D \\ TC_3(T) & \text{if } N \leq T \leq M \\ TC_4(T) & \text{if } 0 < T \leq N \end{cases} \tag{1}$$

where

$$TC_1(T) = \frac{A}{T} + \frac{DTh\rho}{2} + \frac{cI_k\rho}{T}\left[\frac{DT^2 - PM^2}{2}\right] - \frac{sI_eD}{2T}[M^2 - (1-\alpha)N^2] \tag{2}$$

$$TC_2(T) = \frac{A}{T} + \frac{DTh\rho}{2} + \frac{cI_k}{T}\left[\frac{D(T-M)^2}{2}\right] - \frac{sI_eD}{2T}[M^2 - (1-\alpha)N^2] \tag{3}$$

$$TC_3(T) = \frac{A}{T} + \frac{DTh\rho}{2} - \frac{sI_eD}{2T}[2MT - (1-\alpha)N^2 - T^2] \tag{4}$$

$$TC_4(T) = \frac{A}{T} + \frac{DTh\rho}{2} - sI_eD\left[M - (1-\alpha)N - \frac{\alpha T}{2}\right] \tag{5}$$

Since $TC_1(PM/D) = TC_2(PM/D), TC_2(M) = TC_3(M)$ and $TC_3(N) = TC_4(N)$, $TC(T)$ is continuous and well-defined. All $TC_1(T), TC_2(T), TC_3(T), TC_4(T)$ and $TC(T)$ are defined on $T > 0$.

Taking first and second derivatives of the Eqs. (2)-(5), we have the following,

$$TC_1'(T) = -\left[\frac{2A - cI_k\rho PM^2 - sI_eD[M^2-(1-\alpha)N^2]}{2T^2}\right] + D\rho\left(\frac{h+cI_k}{2}\right) \tag{6}$$

$$TC_1''(T) = \frac{2A - cI_k\rho PM^2 - sI_eD[M^2-(1-\alpha)N^2]}{T^3} \tag{7}$$

$$TC_2'(T) = -\left[\frac{2A + cI_kDM^2 - sI_eD[M^2-(1-\alpha)N^2]}{2T^2}\right] + D\left(\frac{h\rho+cI_k}{2}\right) \tag{8}$$

$$TC_2''(T) = \frac{2A + DM^2(cI_k - sI_e) + sI_eD(1-\alpha)N^2]}{T^3} > 0 \tag{9}$$

$$TC_3'(T) = -\left[\frac{2A + sI_eD(1-\alpha)N^2}{2T^2} + \frac{D}{2}(h\rho + sI_e)\right] \tag{10}$$

$$TC_3''(T) = \frac{2A + sD(1-\alpha)N^2I_e}{T^3} > 0 \tag{11}$$

$$TC_4'(T) = \frac{-A}{T^2} + D\left[\frac{h\rho + s\alpha I_e}{2}\right] \tag{12}$$

$$TC_4''(T) = \frac{2A}{T^3} > 0 \tag{13}$$

Eqs. (9), (11) and (13) imply that $TC_2(T), TC_3(T)$ and $TC_4(T)$ are convex on $T > 0$. Eq.(7) implies that $TC_1(T)$ is convex on $T > 0$ when $2A - cI_k\rho PM^2 - sI_eD[M^2 - (1-\alpha)N^2] > 0$. Furthermore, we have $TC_1'(PM/D) = TC_2'(PM/D), TC_2'(M) = TC_3'(M)$ and $TC_3'(N) = TC_4'(N)$. Therefore $TC(T)$ is convex on $T > 0$ when $2A - cI_k\rho PM^2 - sI_eD[M^2 - (1-\alpha)N^2] > 0$.

3.1. Optimal cycle time T^* for the case $M \geq N$

Let $TC_i'(T_i^*) = 0$ for all $i = 1, 2, 3, 4$. We can obtain

$$T_1^* = \sqrt{\frac{2A - cI_k\rho PM^2 - sI_eD[M^2-(1-\alpha)N^2]}{D\rho(h+cI_k)}} \quad \text{if } 2A - cI_k\rho PM^2 - sI_eD[M^2-(1-\alpha)N^2] > 0, \tag{14}$$

$$T_2^* = \sqrt{\frac{2A + cI_kDM^2 - sI_eD[M^2-(1-\alpha)N^2]}{D(h\rho+cI_k)}} \tag{15}$$

$$T_3^* = \sqrt{\frac{2A + sI_eD(1-\alpha)N^2}{D(h\rho+sI_e)}} \tag{16}$$

$$T_4^* = \sqrt{\frac{2A}{D(h\rho+s\alpha I_e)}} \tag{17}$$

Eq.(14) gives the optimal value T^* for the the case when $T \geq PM/D$ so that $T_1^* \geq M$. We substitute Eq.(14) into $T_1^* \geq PM/D$; then we obtain that

$T_1^* \geq PM/D$ if and only if $-2A - cI_kDM^2 + (PM/D)^2D(h\rho + cI_k) + sI_eD[M^2 - (1-\alpha)N^2] \leq 0$.

Eq.(15) gives the optimal value T^* for the the case when $M \le T \le PM/D$ so that $M \le T_2^* \le PM/D$. We substitute Eq.(15) in $M \le T_2^* \le PM/D$; then we obtain that

$T_2^* \le PM/D$ if and only if $-2A - cI_kDM^2 + (PM/D)^2D(h\rho + cI_k) + sI_eD[M^2 - (1-\alpha)N^2] \ge 0.$

and

$$M \le T_2^* \text{ if and only if } -2A + DM^2h\rho + sI_eD[M^2 - (1-\alpha)N^2] \le 0$$

Similarly, Eq.(16) gives the optimal value T^* for the the case when $N \le T \le M$ so that $N \le T_3^* \le M$. We substitute Eq.(16) in $N \le T_3^* \le M$; then we obtain that

$$T_3^* \le M \text{ if and only if } -2A + DM^2h\rho + sI_eD[M^2 - (1-\alpha)N^2] \ge 0$$

and

$$T_3^* \ge N \text{ if and only if } -2A + DN^2[h\rho + s\alpha I_e] \le 0$$

Finally, Eq.(17) gives the optimal value T^* for the the case when $T \le N$ so that $T_4^* \le N$. We substitute Eq.(17) in $T_4^* \le N$; then we obtain that

$$T_4^* \le N \text{ if and only if } -2A + DN^2[h\rho + s\alpha I_e] \ge 0$$

Now, we let

$$\Delta_1 = -2A - cI_kDM^2 + (PM/D)^2D(h\rho + cI_k) + sI_eD[M^2 - (1-\alpha)N^2] \quad (18)$$
$$\Delta_2 = -2A + DM^2h\rho + sI_eD[M^2 - (1-\alpha)N^2] \quad (19)$$
$$\text{and } \Delta_3 = -2A + DN^2[h\rho + s\alpha I_e] \quad (20)$$

Eqs. (18), (19) and (20) imply that $\Delta_1 \ge \Delta_2 \ge \Delta_3$. From the above arguments, we obtain the following Theorem 1.

Theorem 1.

(A) If $\Delta_1 \le 0$, then $TC(T^*) = TC(T_1^*)$ and $T^* = T_1^*$

(B) If $\Delta_1 > 0$ and $\Delta_2 \le 0$, then $TC(T^*) = TC(T_2^*)$ and $T^* = T_2^*$

(C) If $\Delta_2 > 0$ and $\Delta_3 \le 0$, then $TC(T^*) = TC(T_3^*)$ and $T^* = T_3^*$

(D) If $\Delta_3 > 0$ then $TC(T^*) = TC(T_4^*)$ and $T^* = T_4^*$

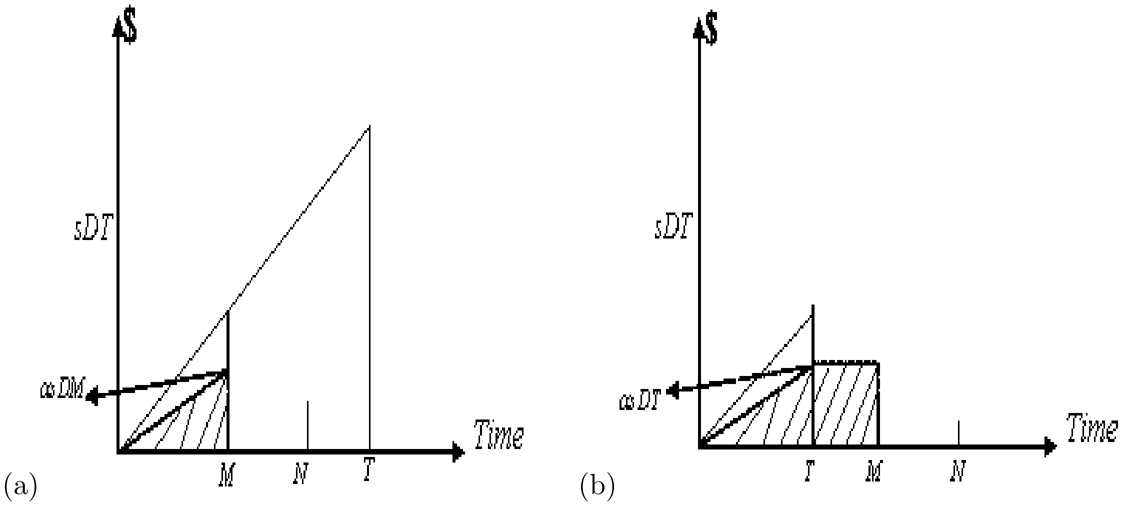


Figure 4: (a) Total amount of interest earned when $M \leq T$, (b) Total amount of interest earned when $T \leq M$

Case II: Suppose $M < N$

1. Annual ordering cost is A/T.

2. Excluding interest charges, annual stock-holding cost is $\frac{DTh\rho}{2}$

3. According to assumption (3), there are three cases that occur in costs of interest payable for the items kept in stock per year.

   (a) For $PM/D \leq T$,

   the annual interest payable

$$= \frac{cI_k\rho}{T}\left[\frac{DT^2 - PM^2}{2}\right]$$

   (b) For $M \leq T \leq PM/D$,

   the annual interest payable

$$= \frac{cI_k}{T}\left[\frac{D(T-M)^2}{2}\right]$$

   (c) $M \geq T$

   the annual interest payable $= 0$

4. According to assumption (4), there are three cases that occur in interest earned per year.

(a) For $PM/D \leq T$ and $M \leq T \leq PM/D$ (see Fig. 4 (a)), the annual interest earned is

$$\frac{sI_eD\alpha M^2}{2T}$$

(b) For $M \geq T$, (see Fig. 4 (b)) the annual interest earned is

$$sI_e\left[\frac{\alpha DT^2}{2} + \alpha DT(M-T)\right]/T = sI_eD[\alpha M - \alpha T/2]$$

The total cost incurred at the retailer, TC(T), is

$$TC(T) = \begin{cases} TC_5(T) & \text{if } PM/D \leq T \\ TC_6(T) & \text{if } M \leq T \leq PM/D \\ TC_7(T) & \text{if } M \geq T \end{cases} \tag{21}$$

where

$$TC_5(T) = \frac{A}{T} + \frac{DTh\rho}{2} + \frac{cI_k\rho}{T}\left[\frac{DT^2 - PM^2}{2}\right] - \frac{sI_eD\alpha M^2}{2T} \tag{22}$$

$$TC_6(T) = \frac{A}{T} + \frac{DTh\rho}{2} + \frac{cI_k}{T}\left[\frac{D(T-M)^2}{2}\right] - \frac{sI_eD\alpha M^2}{2T} \tag{23}$$

$$TC_7(T) = \frac{A}{T} + \frac{DTh\rho}{2} - sI_eD[\alpha M - \alpha T/2] \tag{24}$$

Since $TC_5(PM/D) = TC_6(PM/D)$ and $TC_6(M) = TC_7(M)$, TC(T) is continuous and well-defined. All $TC_5(T), TC_6(T), TC_7(T)$ and $TC(T)$ are defined on $T > 0$. Taking first and second derivatives of the Eqs. (22)-(24), we have the following equations,

$$TC_5'(T) = -\left[\frac{2A - cI_k\rho PM^2 - sI_e\alpha DM^2}{2T^2}\right] + D\rho\left(\frac{h + cI_k}{2}\right) \tag{25}$$

$$TC_5''(T) = \frac{2A - cI_k\rho PM^2 - sI_eD\alpha M^2}{T^3} \tag{26}$$

$$TC_6'(T) = -\left[\frac{2A + cI_kDM^2 - sI_eD\alpha M^2}{2T^2}\right] + D\left(\frac{h\rho + cI_k}{2}\right) \tag{27}$$

$$TC_6''(T) = \frac{2A + DM^2(cI_k - \alpha sI_e)}{T^3} > 0 \tag{28}$$

$$TC_7'(T) = \frac{-A}{T^2} + D\left[\frac{h\rho + s\alpha I_e}{2}\right] \tag{29}$$

$$TC_7''(T) = \frac{2A}{T^3} > 0 \tag{30}$$

Eqs. (28) and (30) imply that $TC_6(T)$ and $TC_7(T)$ are convex on $T > 0$ and Eq.(26) implies that $TC_5(T)$ is convex on $T > 0$ if $2A - cI_k\rho PM^2 - sI_eD\alpha M^2 > 0$. Furthermore, we have $TC_5'(PM/D) =$

$TC_6'(PM/D)$ and $TC_6'(M) = TC_7'(M)$. Therefore, Eq.(21) imply that TC(T) is convex on $T > 0$ when $2A - cI_k\rho PM^2 - sI_eD\alpha M^2 > 0$.

3.2. Optimal cycle time T^* for the case $M < N$

Let $TC_i'(T_i^*) = 0$ for all $i = 5, 6, 7$. We can obtain

$$T_5^* = \sqrt{\frac{2A - cI_k\rho PM^2 - sI_eD\alpha M^2}{D\rho(h + cI_k)}} \quad \text{if } 2A - cI_k\rho PM^2 - sI_e\alpha DM^2 > 0, \tag{31}$$

$$T_6^* = \sqrt{\frac{2A + cI_kDM^2 - sI_eD\alpha M^2}{D(h\rho + cI_k)}} \tag{32}$$

$$T_7^* = \sqrt{\frac{2A}{D(h\rho + s\alpha I_e)}} \tag{33}$$

Eq.(31) gives the optimal value T^* for the the case when $T \geq PM/D$ so that $T_5^* \geq PM/D$. We substitute Eq.(31) in $T_5^* \geq PM/D$; then we obtain that

$$T_5^* \geq PM/D \text{ if and only if } -2A + DM^2(s\alpha I_e - cI_k) + (PM/D)^2D(h\rho + cI_k) \leq 0.$$

Eq.(32) gives the optimal value T^* for the the case when $M \leq T \leq PM/D$ so that $M \leq T_6^* \leq PM/D$. We substitute Eq.(32) in $M \leq T_6^* \leq PM/D$; then we obtain that

$$T_6^* \leq PM/D \text{ if and only if } -2A + DM^2(s\alpha I_e - cI_k) + (PM/D)^2D(h\rho + cI_k) > 0.$$

and

$$M \leq T_6^* \text{ if and only if } -2A + DM^2h\rho + sI_e\alpha DM^2 \leq 0$$

Similarly, Eq.(33) gives the optimal value T^* for the the case when $T \leq M$ so that $T_7^* \leq M$. We substitute Eq.(33) in $T_7^* \leq M$; then we obtain that

$$T_7^* \leq M \text{ if and only if } -2A + DM^2[h\rho + s\alpha I_e] \geq 0$$

Furthermore, we let

$$\Delta_4 = -2A + DM^2(s\alpha I_e - cI_k) + (PM/D)^2D(h\rho + cI_k) \tag{34}$$

$$\Delta_5 = -2A + DM^2(h\rho + s\alpha I_e) \tag{35}$$

Eqs. (34) and (35) imply that $\Delta_4 \geq \Delta_5$. From the above arguments, we obtain the following Theorem 2.

Theorem 2.

(A) If $\Delta_5 \geq 0$, then $TC(T^*) = TC(T_7^*)$ and $T^* = T_7^*$

(B) If $\Delta_5 < 0$ and $\Delta_4 > 0$, then $TC(T^*) = TC(T_6^*)$ and $T^* = T_6^*$

(D) If $\Delta_4 \leq 0$ then $TC(T^*) = TC(T_5^*)$ and $T^* = T_5^*$

4. Particular cases

4.1. Chung and Haung's model (2003)

When $N = 0, M \geq 0, s = c$ and $\alpha = 0$ (it means that the supplier offers full trade credit to his/her retailer but the retailer does not offer trade credit to his/her customer), from Eqs.(2 - 4)

$$TC_8(T) = \frac{A}{T} + \frac{DTh\rho}{2} + \frac{cI_k\rho}{T}\left[\frac{DT^2 - PM^2}{2}\right] - \frac{cI_eDM^2}{2T} \text{if } T \geq PM/D \tag{36}$$

$$TC_9(T) = \frac{A}{T} + \frac{DTh\rho}{2} + \frac{cI_k}{T}\left[\frac{D(T-M)^2}{2}\right] - \frac{cI_eDM^2}{2T} \text{ if } M \leq T \leq PM/D \tag{37}$$

$$TC_{10}(T) = \frac{A}{T} + \frac{DTh\rho}{2} - \frac{cI_e}{T}[DT^2/2 + DT(M-T)] \text{ if } T \leq M \tag{38}$$

From the optimality conditions, we have

$$TC_i^{'}(T_i^*) = 0 \text{ for } i = 8, 9, 10$$

where

$$T_8^* = \sqrt{\frac{2A + DM^2c(I_k - I_e) - PM^2cI_k}{D\rho(h + cI_k)}} \quad \text{if } 2A + DM^2c(I_k - I_e) - PM^2cI_k > 0, \tag{39}$$

$$T_9^* = \sqrt{\frac{2A + DM^2c(I_k - I_e)}{D(h\rho + cI_k)}} \tag{40}$$

$$T_{10}^* = \sqrt{\frac{2A}{D(h\rho + cI_e)}} \tag{41}$$

Eq.(1) is modified as follows:

$$TC(T) = \begin{cases} TC_8(T) & \text{if } PM/D \leq T \\ TC_9(T) & \text{if } M \leq T \leq PM/D \\ TC_{10}(T) & \text{if } M \geq T \end{cases} \tag{42}$$

Eq.(42) is consistent with Eqs.(6 a-c) in Chung and Haung's model (2003). Eqs.(18) and (19) can be modified as

$$\Delta_1 = -2A + \frac{M^2}{D}[P(P-D)h + cI_k(P^2 - D^2) + cI_eD^2] \tag{43}$$

$$\text{and } \Delta_2 = -2A + DM^2(h\rho + cI_e) \tag{44}$$

$$\tag{45}$$

respectively. If we let, $\bar{\Delta}_1 = -2A + \frac{M^2}{D}[P(P-D)h + cI_k(P^2 - D^2) + cI_eD^2]$ and $\bar{\Delta}_2 = -2A + DM^2(h\rho + cI_e)$ then Theorem 1 can be modified as follows.

Theorem 3.

(A) If $\bar{\Delta}_1 \leq 0$ and $\bar{\Delta}_2 < 0$, then $TC(T^*) = TC(T_8^*)$ and $T^* = T_8^*$

(B) If $\bar{\Delta}_1 > 0$ and $\bar{\Delta}_2 < 0$, then $TC(T^*) = TC(T_9^*)$ and $T^* = T_9^*$

(C) If $\bar{\Delta}_1 \geq 0$ and $\bar{\Delta}_2 > 0$ then $TC(T^*) = TC(T_{10}^*)$ and $T^* = T_{10}^*$

The above Theorem has been discussed in Theorem 3 of Chung and Haung's model (2003). Hence, Chung and Haung's model is a particular case of this paper.

4.2. Haung's model (2003)

When $P \to \infty, M \geq N, s = c$ and $\alpha = 0$ (it means that the retailer (under EOQ strategy) also offers full trade credit to his/her customer), we have

$$TC_{11}(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_kD(T-M)^2}{2T} - \frac{cI_eD(M^2-N^2)}{2T} \tag{46}$$

$$TC_{12}(T) = \frac{A}{T} + \frac{DTh}{2} - cI_eD[2MT - N^2 - T^2]/2T \tag{47}$$

$$TC_{13}(T) = \frac{A}{T} + \frac{DTh}{2} - cI_eD(M-N) \tag{48}$$

From the optimality conditions, we have

$$TC_i^{'}(T_i^*) = 0 \text{ for } i = 11, 12, 13$$

where

$$T^*_{11} = \sqrt{\frac{2A + cD[M^2(I_k - I_e) + N^2 I_e]}{D(h + cI_k)}} \tag{49}$$

$$T^*_{12} = \sqrt{\frac{2A + DN^2 I_e}{D(h + cI_e)}} \tag{50}$$

$$T^*_{13} = \sqrt{\frac{2A}{Dh}} \tag{51}$$

Eq.(1) is modified as follows:

$$TC(T) = \begin{cases} TC_{11}(T) & \text{if } M \le T \\ TC_{12}(T) & \text{if } N \le T \le M \\ TC_{13}(T) & \text{if } 0 < T \le N, \end{cases} \tag{52}$$

Eq.(52) is consistent with Eqs.(1 a-c) in Haung's model (2003). Eqs.(18) and (19) can be modified as

$$\Delta_1 = -2A + M^2[h + cI_e] - cDN^2 I_e \tag{53}$$

$$\text{and } \Delta_2 = -2A + DN^2 h \tag{54}$$

$$\tag{55}$$

respectively. If we let, $\bar{\Delta}_3 = \Delta_1$ and $\bar{\Delta}_4 = \Delta_2$ then Theorem 1 can be modified as follows.

Theorem 4.

(A) If $\bar{\Delta}_3 \le 0$ and $\bar{\Delta}_4 < 0$, then $TC(T^*) = TC(T^*_{11})$ and $T^* = T^*_{11}$

(B) If $\bar{\Delta}_3 > 0$ and $\bar{\Delta}_4 < 0$, then $TC(T^*) = TC(T^*_{12})$ and $T^* = T^*_{12}$

(C) If $\bar{\Delta}_3 > 0$ and $\bar{\Delta}_4 \ge 0$ then $TC(T^*) = TC(T^*_{13})$ and $T^* = T^*_{13}$

The above Theorem has been discussed in Theorem 1 of Haung's model (2003). Hence, Haung's model is a particular case of this paper.

4.3. Goyal's model (1985)

When $P \to \infty, N = 0, s = c$ and $\alpha = 0$ (it means that the retailer (under EOQ strategy) would not offer the delay period to his/her customer, that is, one level of trade credit), let

$$TC_{14}(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_k D(T - M)^2}{2T} - \frac{cI_e DM^2}{2T} \tag{56}$$

$$TC_{15}(T) = \frac{A}{T} + \frac{DTh}{2} - cI_e[DT^2/2 + DT(M - T)]/2T \tag{57}$$

From the optimality conditions, we have

$$TC_i^{'}(T_i^*) = 0 \text{ for } i = 14, 15$$

where

$$T_{14}^* = \sqrt{\frac{2A + cDM^2(I_k - I_e)}{D(h + cI_k)}} \tag{58}$$

$$T_{15}^* = \sqrt{\frac{2A}{D(h + cI_e)}} \tag{59}$$

Eq.(1) is modified as follows:

$$TC(T) = \begin{cases} TC_{14}(T) & \text{if } M \leq T \\ TC_{15}(T) & \text{if } 0 \leq T \leq M \end{cases} \tag{60}$$

Eq.(60) is consistent with Eqs.(1) and (4) in Goyal's model (1985) respectively. Eq.(18)) can be modified as $\Delta_1 = -2A + M^2[h + cI_e]$. If we let, $\bar{\Delta} = \Delta_1$ then Theorem 1 can be modified as follows.

Theorem 5.

(A) If $\bar{\Delta} < 0$ then $T^* = T_{14}^*$

(B) If $\bar{\Delta} > 0$ then $T^* = T_{15}^*$

(C) If $\bar{\Delta} = 0$ then $T^* = T_{14}^* = T_{15}^* = M$

The above Theorem has been discussed in Theorem 1 of Goyal's model (1985). Hence, Goyal's model (1985) is a particular case of this paper.

5. Numerical Examples

In order to evaluate the proposed model, we have designed 27 numerical problems for different parameters of α, N and s when $M \geq N$ and $M < N$. By using Theorems 1 and 2, we have obtained optimal solutions.

5.1. When $M \geq N$

For convenience, the following set of input values for various inventory parameters are selected randomly. Let $P = 3000$; D= 2000; $A = 100$; $c = 14$; $h = 7$; I_k= 0.1; $I_e = 0.2$; $M = 0.1$. The optimal solutions are shown in Table 1.

Table 1: Optimal solutions when $M \geq N$

α	N	s	Δ_1	Δ_2	Δ_3	Theorem	T^*	Q^*	TC^*
0.2	0.03	10	< 0	< 0	< 0	1(A)	0.1631	326.1025	913.0871
		20	> 0	< 0	< 0	1(B)	0.1435	287.0042	791.4825
		30	> 0	< 0	< 0	1(B)	0.1250	249.9714	653.2267
	0.06	10	< 0	< 0	< 0	1(A)	0.1677	335.4315	939.2082
		20	< 0	< 0	< 0	1(A)	0.1518	303.5975	850.0729
		30	> 0	< 0	< 0	1(B)	0.1382	276.3538	751.7209
	0.09	10	< 0	< 0	< 0	1(A)	0.1752	350.4283	981.1993
		20	< 0	< 0	< 0	1(A)	0.1679	335.7720	940.1617
		30	< 0	< 0	< 0	1(A)	0.1602	320.4461	897.2491
0.5	0.03	10	< 0	< 0	< 0	1(A)	0.1625	324.9176	909.7692
		20	> 0	< 0	< 0	1(B)	0.1425	284.9812	783.9298
		30	> 0	< 0	< 0	1(B)	0.1232	246.4752	640.1739
	0.06	10	< 0	< 0	< 0	1(A)	0.1654	330.7999	926.2397
		20	> 0	< 0	< 0	1(B)	0.1475	294.9576	821.1751
		30	> 0	< 0	< 0	1(B)	0.1317	263.4930	703.7073
	0.09	10	< 0	< 0	< 0	1(A)	0.1702	340.3779	953.0582
		20	> 0	< 0	< 0	1(A)	0.1572	314.4156	880.3636
		30	> 0	< 0	< 0	1(B)	0.1448	289.6426	801.3325
0.8	0.03	10	< 0	< 0	< 0	1(A)	0.1619	323.7283	906.4392
		20	> 0	< 0	< 0	1(B)	0.1415	282.9437	776.3232
		30	> 0	< 0	< 0	1(B)	0.1215	242.9286	626.9333
	0.06	10	< 0	< 0	< 0	1(A)	0.1631	326.1025	913.0871
		20	> 0	< 0	< 0	1(B)	0.1435	287.0042	791.4825
		30	> 0	< 0	< 0	1(B)	0.1250	249.9714	653.2267
	0.09	10	< 0	< 0	< 0	1(A)	0.1650	330.0216	924.0606
		20	> 0	< 0	< 0	1(B)	0.1468	293.6470	816.2822
		30	> 0	< 0	< 0	1(B)	0.1306	261.2880	695.4753

5.2. When $M < N$

Let P = 6000; D= 5000; A = 43; c = 10; h = 10; I_k= 0.1; I_e = 0.2; M = 0.05. The optimal solutions are shown in Table 2.

The following results are observed from Tables 1 and 2

(1) For fixed value of N and s, the larger the value of α, the smaller the value of the optimal cycle time and the lower the value of the annual total cost.

(2) For fixed α and s, the larger the value of N, the larger the value of the optimal cycle time and the higher the value of the annual total cost when $M \geq N$; the optimal cycle time and the annual total cost will be independent of N when $M < N$.

(3) Finally, for fixed α and N, the larger the value of s, the smaller value of the optimal cycle time and the smaller the value of the annual total cost.

To study the effects of α, N and s on the optimal order quantity Q^* and on the optimal total cost $TC(T^*)$, there are some managerial phenomena from Tables 1 and 2: (1) When the unit selling price is increasing, the retailer will order less quantity to take the benefits of the trade credit more frequently. (2) When the customer's trade credit period offered by the retailer is increasing, the retailer will order more quantity to accumulate more interest to compensate the loss of interest earned when longer trade credit period is offered to his/her customer under the condition of M $\geq$ N. (3) When the customer's fraction of the total amount owed payable at the time of placing an order offered by the retailer is increasing, the retailer will order less quantity and increase order frequency. The retailer can accumulate more interest under higher order frequency and higher customer's fraction of the total amount owed payable at the time of placing an order offered by the retailer.

6. Conclusions and Future Research

The results in this paper not only provide a valuable reference for decision-makers in planning and controlling the inventory but also provide a useful model for many organizations that use the decision rule to improve their total operation cost. Here, we develop an EPQ inventory model that investigates retailer's decision making right in a supply chain under some realistic features. First, the supplier provides his retailer a full trade credit period and the retailers also adopt the partial trade credit option to their customers to stimulate high sales. Second, interest charge rate is not necessarily higher than the interest earned rate. These assumptions are consistent with economic senses. We develop two effective and easy-to-use theorems to help the decision maker to find the optimal replenishment policy. Theorem 1 gives the decision rule of the optimal cycle time when $M \geq N$ after computing the numbers Δ_1, Δ_2 and Δ_3. Theorem 2 does the decision rule of the optimal cycle time when $M < N$ after computing the numbers Δ_4 and Δ_5. Then we deduce Chung and Haung's model (2003), Haung's model (2003) and Goyal's model (1985) as particular cases of this paper. Numerical examples are given to illustrate all effective theorems and obtained a lot of managerial insights.

In practice, the contributions of this paper and the approach we considered to solve the problem are significant because the retailer has to decide whether it is worthwhile to alter the regular ordering pattern to exploit other opportunities and assess their monetary impact to find the optimal ordering policy under realistic conditions linking marketing as well as operations management concerns. Finally, this paper brings attention into the trade credit that is of major importance in the operations of enterprises in many economics. The proposed model can be extended in several ways. For instance, we may extend this for perishable items. In addition, we could consider the probabilistic demand with shortages and quantity discounts.

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Table 2: Optimal solutions when $M < N$

α	N	s	Δ_4	Δ_5	Theorem	T^*	Q^*	TC^*
0.1	0.06	10	< 0	< 0	2(C)	0.0940	470.0097	861.6844
		30	< 0	< 0	2(C)	0.0911	455.2721	834.6656
		50	< 0	< 0	2(C)	0.0880	440.0413	806.7424
	0.08	10	< 0	< 0	2(C)	0.0940	470.0097	861.6844
		30	< 0	< 0	2(C)	0.0911	455.2721	834.6656
		50	< 0	< 0	2(C)	0.0880	440.0413	806.7424
	0.1	10	< 0	< 0	2(C)	0.0940	470.0097	861.6844
		30	< 0	< 0	2(C)	0.0911	455.2721	834.6656
		50	< 0	< 0	2(C)	0.0880	440.0413	806.7424
0.5	0.06	10	< 0	< 0	2(C)	0.0708	440.0413	806.7424
		30	> 0	< 0	2(B)	0.0520	354.1956	649.3587
		50	< 0	< 0	2(C)	0.0880	259.8076	442.8203
	0.08	10	< 0	< 0	2(C)	0.0880	440.0413	806.7424
		30	< 0	< 0	2(C)	0.0708	354.1956	649.3587
		50	> 0	< 0	2(B)	0.0520	259.8076	442.8203
	0.1	10	< 0	< 0	2(C)	0.0880	440.0413	806.7424
		30	< 0	< 0	2(C)	0.0708	354.1956	649.3587
		50	> 0	< 0	2(B)	0.0520	259.8076	442.8203
0.9	0.06	10	< 0	< 0	2(C)	0.0816	407.8770	747.7745
		30	> 0	> 0	2(A)	0.0493	246.6760	393.1772
		50	> 0	< 0	2(A)	0.0402	200.7797	108.3495
	0.08	10	< 0	< 0	2(C)	0.0816	407.8770	747.7745
		30	< 0	> 0	2(A)	0.0493	246.6760	393.1772
		50	> 0	> 0	2(A)	0.0402	200.7797	108.3495
	0.1	10	< 0	< 0	2(C)	0.0816	407.8770	747.7745
		30	> 0	> 0	2(A)	0.0493	246.6760	393.1772
		50	> 0	> 0	2(A)	0.0402	200.7797	108.3495

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